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The Babylonian Astronomy. By R. H. M. Bosanquet and Prof. A. H. Sayce.

No. 3. *The Venus Tablet.*

The name of the planet *Venus*, in the writings of ancient Babylon is known to us with as great certainty as any word in the inscriptions. In the small list of words preserved by Hesykhius, there is the word δελεφάτ, which he gives as the name of the Babylonian Venus. The word Dilbat occurs in the inscriptions as the name of a planet, and corresponds generally, as to its place in the lists, with *Venus*. The planet Dilbat is further identified with the goddess Istar,* and sometimes also with the goddess Anunit.† Istar in turn, regarded as a star, was called Nin-śi-anna, "Lady of the defences of heaven." To show clearly our authority for this statement, the following words are written down, exactly representing two corresponding lines of an inscription in two parallel columns.

W. A. I. ii. 59, 20 (2).

Divine Ninśianna.	Divine Istar; a star.

We thus arrive at the name Ninśianna as a name commonly used for Venus.

The tablet which forms the subject of the present communication speaks of the "divine Ninśianna" throughout. The tablet is a good deal broken, but a considerable number of observations survive. The following is the translation of the

* See *Trans. Soc. Bibl. Arch.* iii. 197; also *W. A. I.* (*Western Asiatic Inscriptions*), ii. 49, 11.

† *W. A. I.* ii. 49, 12.

tablet. For the text see *W. A. I.* iii. 63 and *Trans. Soc. Bibl. Arch.* iii. 316.

OBVERSE.

LINE.

1. . . . . .
2. . . . . Venus disappeared, and the 2nd day Venus . . . .
3. . . . . for a year are. A destruction takes place.
4. In the month Tammuz, the 25th day, Venus at sunset disappeared.
5. The 7th day* in heaven it is visible; and in the month Ab, the 2nd day, Venus
6. at sunrise is seen. Waters in the land are. Beating of . . . .
7. In the month Adar, the 25th day, Venus at sunrise disappeared.
8. For a year service continues. Gold . . . .
9. In the month (Sivan) the 11th day† Venus at sunset disappeared. The 9th month, the 4th day, in heaven it was visible; and
10. in the month Adar, the 15th day, at sunrise it is seen. King to king peace sends.
11. In the month Marchesvan, the 10th day, Venus at sunrise disappeared. The 2nd month, the 6th day, in heaven it appeared.‡
12. In the month Tebet, the 16th day, at sunset it is seen. The crops of the land are prosperous.
13. In the month Elul, the 26th day, Venus at sunset disappeared. The 11th day, in heaven it appeared.
14. In the second Elul, the 7th day, at sunset it is seen. The heart of the land is good.
15. In the month Nisan, the 9th day, Venus . . . . disappeared. The 5th month, the 16th day, in heaven (it reappeared; and)
16. In the month Elul, the 25th day, at sunset it is seen. The heart of (the land is good).
17. In the month Iyyar, the 5th day, Venus at sunset is seen . . . . in heaven it appeared; and
18. At sunrise it is seen. The crops of the land are good . . . . the 10th day at sunrise . . . .
19. At sunrise it disappeared. The 15th day in heaven . . . . In the month Sebat, the 11th day at sunset . . . .
20. In the month Ab, § the 10th day, Venus at sun . . . . The 1st month,‖ the 16th day . . . .
21. In the month Marchesvan, the 26th day, at sunset it is seen . . . ; Rains in the land are . . . .

* L. 5. The comparison of the interval of 7 days with the preceding and following observations shows that the month was here supposed to be 30 days

† L. 9. It is established, by comparison of passages in which a number of months and days is mentioned, that in all such cases reference is made to the interval between two observations, and not to the year-number of the month. Here the interval permits us to restore the month of the first observation, which is Sivan; it will appear from the subsequent discussion that this date belongs to the appearance at sunset, not to the disappearance.

‡ L. 13–14. Month of 30 days.

§ L. 20–21. Mistake in the month-number of one of the three entries.

‖ Or "after one month and 16 days."

22. In the month . . . . the 20th day, Venus at sunset disappeared. The 2nd month, the 16th day . . . .

23. In the month . . . . the 4th day, at sunset it is seen. Rains in the land are.

24. In the month . . . . the 6th day, Venus at sunset disappeared The 15th day . . . .

25. In the month . . . . the 20th day, at sunset it is seen. Rains in heaven, floods in the channels are . . . .

26. In the month Adar, the 26th day, Venus at sunrise disappeared. The 3rd month, the 9th day . . . .

27. In the month Sivan, the 20th day, at sunset it is seen. The forces of barbarian soldiery are collected . . . .

28. In the month Adar, the 11th day, Venus at Sun . . . . (disappeared). The 4th day in heaven . . . .

29. The crops of the earth flourish. The heart of the land is good.

30 . . . . wind, rain, snow, ...*

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31. †In the month Nisan, the 2nd day, Venus at sunrise is seen. Deserts in the land are.

32. Up to the 6th day of the month Chisleu at sunrise it is fixed.‡ The 7th day of Chisleu it disappears, and after 3 months in heaven

33. It appears; and the 8th day of the month Adar, Venus at sunset rises and king to king hostility sends.

34. In the month Iyyar, the . . . . day, Venus at sunset is seen Hostile bands in the land are.

35. (Up to the 7th day of) the month Tebet at sunset it is fixed.§ The 8th day of Tebet it disappears; and

36. (The 7th day in heaven) it appears; and the 15th day of Tebet, Venus

37. At sunrise rises; and the crops of the land are flourishing, the heart of the land is good.

38. (In the month Sivan, the 4th day) Venus at sunrise is seen. An inundation in the land.

39. (Up to the 8th day of the month Sebat) at sunrise it is fixed. The 8th day of Sebat it disappears; and

40. After (3) months in heaven appears; and the 9th day of the month Iyyar, Venus

41. At sunset rises; and hostile bands in the country are.

42. In the month Tammuz, the 5th day, Venus at sunset is seen. Hostile bands in the land are. The crops of the land flourish.

43. Up to the 9th day of the month Adar at sunset it is fixed. The 10th day of Adar it disappears; and

44. The 7th day in heaven it appears; and the 7th day of the month Adar, Venus

45. At sunrise rises; and king to king hostility sends.

* L. 30. Here the tablet marks the end of a paragraph.

† L. 31. Change of style and grammatical usage. Probable beginning of the spurious part. See *post.*

‡ L. 32. *I.e.* seen.

§ L. 35. *I.e.* seen.

REVERSE.

1. In the month Ab, the 6th day, Venus at sunrise is seen. Rains in heaven are. A beating takes place.
2. Up to the 10th day of the month Nisan, at sunrise it is fixed. The 11th day of Nisan it disappears; and
3. After 3 months in heaven it is seen; and the 11th day of the month Tammuz, Venus at sunset
4. Rises; and hostile bands in the land are. The crops of the land are prosperous.
5. In the month Elul, the 7th day, Venus at sunset is seen. The crops of the land flourish. The heart of the land is good.
6. Up to the 11th day of the month Iyyar, at sunset it is fixed. The 12th day of Iyyar it disappears; and
7. After 7 days in heaven it reappears; and the 9th day of the month Iyyar, Venus
8. At sunrise rises; and hostile bands in the country are.
9. In the month Tisri, the 8th day, Venus at sunrise is seen. Hostile bands in the land are. The crops of the land flourish.
10. Up to the 12th day of the month Sivan at sunrise it is fixed. The 13th day of Sivan it disappears; and
11. After 3 months in heaven it reappears; and the 13th day of the month Elul, Venus
12. At sunset rises; and the crops of the land are prosperous; the heart of the land is good.
13. In Marchesvan, the 9th day, Venus at sunset is seen. The land a strong woman seizes.
14. Up to the 13th day of the month Ab, at sunrise it is fixed. The 15th day of Ab, it disappears and
15. The 7th day in heaven it reappears; and the 11th day of the month Tammuz, Venus
16. At sunrise rises; and hostilities in the land are. The crops of the land flourish.
17. In the month Chisleu, the 10th day, Venus at sunrise is seen. Want of corn and barley in the land is.
18. Up to the 14th day of the month Ab, at sunrise it is fixed. The 15th day of Ab, it disappears and
19. After 3 months in heaven it reappears; and the 15th day of Marchesvan, Venus
20. At sunset rises; and the crops of the land are prosperous.
21. In the month Tebet, the 11th day, Venus at sunset is seen. The crops of the land flourish.
22. Up to the 15th day of the month Elul, at sunset it is fixed. The 16th day of Elul it disappears; and
23. After 7 days in heaven it reappears; and the 23rd day of the month Elul, Venus
24. At sunset rises, and the crops of the land are prosperous.
25. In the month of Sebat, the 12th day, Venus at sunrise is seen. The crops of the land flourish.

26. Up to the 16th day of the month Tisri, at sunrise it is fixed. The 17th day of Tisri it disappears; and
27. After 3 months in heaven it reappears; and the 17th day of the month Tebet, Venus
28. At sunset rises; and . . . .
29. In the month Adar, the 13th day, Venus at sunset is seen. The king . . . .
30. Up to the 17th day of Marchesvan, at sunset it is fixed. The 18th day of Marchesvan, Venus
31. The 7th day in heaven reappears; and the 25th day of Marchesvan, Venus
32. At sunset rises; and the land a strong woman seizes.
33. Twelve collections of observations of the risings of Venus in parallel columns (Accadian and Assyrian) of Babylon.
34. In the 2nd Elul, the 1st day, Venus at sunset disappears.*
35. The 15th day in heaven it is seen; and in the 2nd Elul, the 17th day, Venus
36. At sunrise is seen. A prodigy in the land is; in the palace . . . .
37. In the month Sivan, the 25th day, Venus at sunrise disappears . . .
38. After 2 months, the 6th day, in heaven it is seen; and in the month of Elul, the 24th day,
39. Venus at sunset is seen. The heart of the country is good.
40. In the month Nisan, the 26th day, Venus at sunset disappears; and
41. After 7 days in heaven it reappears; and the month Iyyar, the 3rd day, Venus
42. At sunrise is seen. Hostile bands in the land are. The crops of the land are prosperous.
43. . . . . Venus at sunrise disappears, and
44. . . . . in the month Adar, the 28th day, Venus
45. . . . . king to king an ambassador sends

The two following lines, which complete the tablet, are broken off.

It will be seen that there are no year-dates in this tablet; consequently it might be supposed that no further information is obtainable from it. And, indeed, it has been supposed that the observations were probably isolated, and the whole practically of no interest.

Tested, however, by a simple application of the synodical period, we find that the observations link themselves into a certain number of numerical schemes: so that portions of the tablet in question almost certainly refer to continuous series of phenomena. And the different portions of the tablet present differences in the numerical schemes according to which they are constituted, the discussion of which leads to singular results.

We confine ourselves in the first instance to the numbers of

* L. 34. Here begins a new paragraph.

months concerned. This affords a rough test of the synodic period, without involving hypotheses as to detail.

It will be desirable to recall approximate values of the synodic periods of *Venus* and *Mercury*. For *Venus* the number is about 514^d, say 19·2 mean months, or about 19·7 lunar months; for *Mercury*, less than 4 months. The value in the case of *Mercury* is only needed to assure us that no confusion has taken place with that planet.

We will now form an analysis of the dates of the tablet, mentioning in each case only the characteristic number of the month mentioned. These numbers are:—

1 Nisan.	5 Ab	9 Chisleu.
2 Iyyar.	6 Elul.	10 Tebet.
3 Sivan.	7 Tisri.	11 Sebat.
4 Tammuz.	8 Marchesvan.	12 Adar.

Lines of Tablet.	Sunset. Appears.	Is seen.	Disappears.	Sunrise. Appears.	Is seen.	Disappears.
			Obverse.			
4–7			4	5		12
10–11	3		....	12		8
11–15	10		6	7 (2nd Elul)		1
15–19	6		2	....		....
19			11 (?)	. . . .		5 (?)
21–22	8					....
24–26		....	....	....		12
27	3		[12]			
31–32				1		9
33–39	12	2	10	10	3	11
40–45	2	4	12	12		
			Reverse.			
1–2					5	1
3–10	4	6	2			3
11–18	6	8	[5]	4		5
19–26	8	10	6	[6]	11	7
27–32	10	12	8	[8]		
33		12 collections of observations, &c.				
34–37			7	7		3
38–43	6		1	2		[ ]
44	12					

(Two lines broken off.)

We have first to consider whether the numbers afford proof or probability of the observations being consecutive. This we answer at once in the affirmative.

The following scheme represents the differences of the month-numbers in the successive rows. This shows all the synodic periods of which both ends are given. It affords a means of appreciating the differences of character in the different parts of the tablet.

Differences of successive Rows.

Lines of Tablet.	Sunset. Appears.	Sunset. Is seen.	Sunset. Disappears.	Sunrise. Appears.	Sunrise. Is seen.	Sunrise. Disappears.
			Obverse.			
4– 7				19		20
0–11	19			19		19
11–15	18		20			
15–19						
19						
21–22						
31–32				21		26
33–39	26	26	26	26	26	26
40–45 and Reverse 1–2	26	26	26	26	26	26
3–10	26	26	27	26	26	26
11–18	26	26	25	26	26	26
19–26	26	26	26	26		
27–32						
			12 collections of observations, &c.			
34–37				18		19
38-43	18					
44						

We see at once that the successions are of such a character as to leave no doubt of the continuity of portions of the scheme.

The first 20 lines contain numbers leading to 6 synodic periods between 18 and 20 months, and one of 17 months. The one of 17 is altered to 19, on noticing that it is preceded by a 2nd Elul, and admitting a second intercalary month. This and the remainder are compatible with the synodic period of *Venus*.

Then follow ten lines too imperfect for analysis.

In l. 31 there is one date which combined with the first corresponding number of the next part gives a period of 21 months; this is within the limits of error.

Then follows a sequence of numbers, commencing in line 32, and continuing uninterruptedly down to line 32 of the reverse.

Corresponding figures in successive lines differ by 2 throughout. Six phases are enumerated in every period, without a single omission; and every month-number, except one, throughout six synodic periods falls in exactly with an obvious numerical scheme. The synodic period thus definitely announced is 26 months. No assumption can get rid of the excess of at least 6 months over the period of *Venus*; and it is clear, not only that this portion of the tablet is a fabrication, but also that it is a fabrication by some person wholly unacquainted with the phenomena. There can be little doubt that the scribe, finding this portion of the ancient Babylonian tablet illegible, fulfilled his duty by reconstructing it out of his inner consciousness, for the benefit of the great astronomical collection of Sargon of Aganè. We may note that the part spoken of as spurious differs in style, grammar, &c., from the rest. But it was picked out by one of us from the numbers only, without any knowledge of these peculiarities.

The next line (33 reverse) is

"Twelve collections of observations of the risings of Venus in parallel columns (Accadian and Assyrian) from Babylon."

It is supposed that this is the colophon of the previous portion of the tablet, which is arranged in twelve well-defined short paragraphs or sentences. The text is not bilingual. We therefore suppose it to be copied or made up from an earlier bilingual text in the library of Babylon; the Accadian original being probably earlier still.

In what is preserved of the remaining lines of the tablet, we have three synodic periods of 18, 19, and 18 months respectively, the last observation of the last pair being incomplete. These are within the limits reconcilable with the true period of *Venus* by means of intercalary months. And we are bound to admit, consequently, that the observations bear *primâ facie* the stamp of genuineness.

Any further examination of the tablet may consequently be confined to the first 21 lines, and to what follows the line "Twelve collections of observations."

We proceed to consider somewhat more in detail the complete observations in the first 20 lines. The following is a scheme of these observations, in which the month-number is replaced by the number of complete months. Thus the 9th day of Nisan is written $0^m\ 9^d$.

	Sunset.		Sunrise.	
Lines.	Appears.	Disappears.	Appears.	Disappears.
	m d	m d	m d	m d
4– 7		3 25	4 2	11 25
9–11	2 11	—	11 15	7 10
11–15	9 16	5 26	6 7 (I)	0
15–19	5 25	1 5	—	—

The observations of the next few lines are too imperfect for certain discussion, though they might possibly be reconstructed.

The fact that the months whose length is accidentally given (see notes) are of 30 days, appears to point to a different calendar from that of the lunar months. We have calculated the synodic periods both on the assumption of months of 30 days, and of lunar months; the results are not in either case so completely in accordance as to decide the question. It is not improbable that the months were arbitrarily arranged from time to time, partly according to one rule, partly according to the other.

The observation to which (I) is affixed is in the month 2nd Elul—an intercalary, apparently foreign to the regular calendar, in which the intercalary was at the end of the year, and called *Veadar*.*

The synodic period of *Venus* is, according to our calculation, $583^{d}.919864$. For the present purpose it will be sufficient to consider it as 584^{d}.

The synodic periods deduced singly from the differences of the above observations are:—

y m d	y m d	y m d	y m d
		1 7 13	1 7 15
1 7 5	—	1 6 22 (I)	1 4 29
1 8 9	1 7 9		

(1 y. = 12 months simply.)

	d	l.y. l.m. d.
Months of $29\frac{1}{2}$ days	584 =	1 7 $26\frac{1}{2}$
	y	m d
Months of 30 days	584 =	1 7 14

(1 l.y. = 1 lunar year = 354d. 1y. = 360d.)

The first two numbers present coincidences which leave no doubt that during the observations of lines 4–11, the months were counted as of 30 days each.

The subsequent periods present deviations which seem to indicate first that intercalaries were introduced; also either that the text is corrupt (which is very probable) or that lunar months were returned to in the course of the series. The number $1^{y}\ 4^{m}\ 29^{d}$ is preceded by one intercalary month shown in an observation and denoted by (I); there also appears to be another intercalary month before line 12, as is obvious from the number $1^{y}\ 6^{m}\ 22^{d}$; and the two together would raise the $1^{y}\ 4^{m}\ 29^{d}$ to nearly $1^{y}\ 7^{m}$, which is within limits of calendar error. The number $1^{y}\ 8^{m}\ 9^{d}$ is not so easily accounted for. $1^{y}\ 7^{m}\ 9^{d}$ is within the limits of error, if it were not for the intervening intercalary. This makes the last number really $1^{y}\ 8^{m}\ 9^{d}$, some 12^{d} greater than is consistent with lunar months. Further, we

* See Norris's *Assyrian Dictionary*, i. 50.

must notice that appearances of the planet may probably have been delayed by cloudy weather. Therefore it is possible that on this ground certain of the observations may not accurately correspond to the phases in the synodic scheme.

We will now form the differences corresponding to two synodic periods. These are:—

y m d	y m d	y m d	y m d
	3 2 1	3 2 5	3 0 14
3 3 14	—		

	d	d	l.y.	l.m.	d.
Months of $29\frac{1}{2}$		$2 \times 584 =$	3	3	$23\frac{1}{2}$
	d		y	m	d
Months of 30		$=$	3	2	28

It is clear from the first two numbers that there must, in any case, have been an intercalary month before line 12, as before suggested. This would cause the above numbers to assume the following form:—

y m d	y m d	y m d	y m d
	3 3 1	3 3 5	3 2 14
3 5 14	—		

The first two are within the limits of calendar and other error, especially if we suppose that lunar months were employed during a part of the period. The third is scarcely admissible unless we assume a third intercalary immediately preceding it. In any case, the fourth number is irreconcilable.

One pair of numbers gives us 3 synodic periods—the first and last of the above observations. The resulting number is:—

	y	m	d
	4	9	10

	d	d	l.y.	l.m.	d
Months of $29\frac{1}{2}$		$3 \times 584 =$	4	11	$20\frac{1}{2}$
	d		y	m	d
Months of 30		$=$	4	10	12

It must be remembered that one intercalary is actually mentioned in the observations, and there is sufficient evidence of another. Taking these into account we find for the corrected interval given by the two extreme observations:—

y	m	d
4	11	10

counting the year at 12 months.

This coincides substantially with the number corresponding to lunar months.

We have decided for the present to content ourselves with presenting the above comments on this remarkable series of numbers. They sufficiently indicate the nature of the results

obtainable, without professing to give a final treatment of the subject. We have applied various corrections, and constructed various hypothetical calendars; but the results have not been such as to encourage us to produce them. It is possible that when more is known of the history of this ancient period the difficulties of the text may be more easily dealt with.

We may give one illustration of the application of a correction which seems at first sight feasible and even necessary, and yet has been definitely abandoned, at least for the present.

On the reappearance of *Venus* from the region of the Sun, or on its disappearance into the Sun's rays, it would seem that the vertical distance of *Venus* above the Sun, as the latter dips below the horizon, should determine the critical position when the planet is just seen. And according to the time of year, a definite vertical distance such as 12° corresponds to very different distances along the ecliptic. In order to obtain determinations of the synodic period, it would therefore be necessary to apply corrections which should reduce the dates to accurately corresponding configurations. Thus when the ecliptic is more inclined to the vertical, the appearance is later and the disappearance sooner by a certain amount depending, for a particular phase, on the inclination of the ecliptic. Although we cannot tell precisely when the equinox fell, yet assuming that it was approximately at the beginning of the year, it is easy to calculate rough values of these corrections. We have calculated them for each month, and examined the effect of their application. Their amount is considerable; and they appear to throw the observations into confusion. They spoil the synodic periods that do exist, and do not improve materially those that are faulty.

The explanation of this appears to be, that the visibility of *Venus* depends more on its (crescent) phase than on the vertical distance from the Sun in the horizon. If this be admitted, the distance from the Sun in the ecliptic becomes the only point of importance, and the substantial correctness of synodic periods derived directly from the observations is to some extent explained.

The observations which occupy the last lines of the tablet are as follows:—

	Sunset.		Sunrise.	
Line.	Appears. m d	Disappears. m d	Appears. m d	Disappears. m d
34–37		6 1	6 15 (I)	2 25
38–43	5 24	0 26	1 3	—
44	11 28 (? Imperfect.)			

Differences y m d and y m d
1 6 25 and 1 6 18

But on account of the intercalary 2nd Elul the corresponding synodic periods are—

y m d and y m d
1 7 25 and 1 7 18

One of these numbers would decide for the lunar months; the other for the 30 days. Unfortunately, we cannot decide between them.

As to the interval between the disappearance at sunset and the appearance at sunrise.

In two cases this interval is specified to be 7 days (lines 5, 41), but not elsewhere throughout the genuine portion. In line 13 this interval is specified as of 11 days, and in line 35 (rev.) as of 15 days. Throughout the portion which we consider not genuine, this interval is uniformly stated to be 7 days. A slight calculation shows that if we suppose *Venus* to be visible at sunrise or sunset when 12° from the Sun in the ecliptic, this interval should consist of 15 days nearly. It is possible that *Venus* may be visible in tropical regions when nearer to the Sun than this. But in order that the interval may be only 7 days, it is necessary that *Venus* should be visible when only 6° from the Sun in the ecliptic. We beg to call the attention of astronomers in tropical countries to the question at what distances from the Sun *Venus* can be actually seen at sunrise or sunset. But in the mean time we consider the interval of 7 days as highly doubtful, and casting a doubt upon an observation in each case where it occurs. Its uniform occurrence in the spurious part of the tablet is in accordance with this opinion.

As to the interval between the disappearance at sunrise and the appearance at sunset, calculation based on a distance of 12° from the Sun gives 54 days nearly. The numbers from the text are 76 days, 66 days, and 5 months 16 days. The two first numbers accord pretty well with what should happen, the distance from the Sun being taken as greater than 12°, since *Venus* is in the least bright portion of its course. The last number cannot represent an observation.

The number 89 days also occurs in the last lines of the tablet.

The only remaining point worth noticing is that if a tablet such as this were discovered, with first-hand observations, and any king's name, or mark of historical connection which would fix the date within, say, 100 years, it might be possible to fix the date more closely by means of our knowledge of the motions of *Venus*. Nothing of the kind is actually possible with this tablet, as there is no clue whatever to the age of the observations. But, with a view to possible discoveries, it may be well to state shortly the considerations involved.

It is well known that 8 years are nearly equal to 13 revolutions of *Venus*. If this were exactly true, the phases of *Venus* would go through a cycle in 8 years, forming 5 synodic revolutions, and the cycle would repeat itself in successive periods of 8 years. Since, however, the commensurability of mean motions is not exact, the dates of the phases at given points of the cycle do not remain fixed, but shift slowly. If then *Venus* has a definite phase on a given day of the year, the only possible dates are given by the shifting cycle, and it will appear that two such

possible dates for a given day are separated by an interval of 235 years nearly. If, therefore, we can assign the date approximately within, say, half one of these intervals, it is quite possible that we might be able to fix the date more closely by means of the observations of *Venus*.

The numbers are obtained as follows:—

(Herschel's *Outlines of Astronomy*, table at the end.)

Sidereal year	$365^{d}\cdot256361 = a$
Sidereal rev. of Venus	$224\cdot700787 = b$

in mean solar days. Whence synodic period in days

$$= \frac{ab}{a-b}$$

$$= 583^{d}\cdot919864$$

8 years (sidereal)	$= 2922^{d}\cdot05089$
5 synodic periods	$= 2919\cdot59932$
deviation of cycle	$= 2\cdot45157$

To examine the recurrence of the synodic revolutions in the cycle suppose the cycle to be true. Then the synodic period would be in mean months

$$\frac{96^{m}}{5} = 19^{m}\cdot2$$

and the synodic periods from the beginning of the cycle are—

y	m
0	0·0
1	7·2
3	2·4
4	9·6
6	4·8

after which the cycle begins again.

Now, suppose that at a given epoch *Venus* is in a given phase, and that this takes place at the zero of the cycle. Then (neglecting the eccentricity of the orbits and other corrections) *Venus* can only be in the given phase at the other times of the year shown in the column of months. For instance, this cannot occur between the month 0 and the middle of the month 3. But for ancient and future times the cycle is displaced to the extent above indicated.

The quantity which appears above as 2·4 mean months is, when calculated more accurately, $72^{d}\cdot070645$. Dividing this by

the deviation in one cycle, and multiplying by 8^y, we find for the period of change through one interval of the cycle, $235^y \cdot 182$ (sidereal years).

It would be quite possible in this way to calculate the dates at which the observations of this tablet could have been made; but a conjectural element enters into the reconstruction of the calendar of the observations. And as there is nothing to associate these observations with historical dates, there is no possibility of a real contribution to ancient history in this case.

It is, however, amply proved that these observations do actually refer to *Venus*, except the portion which we have excepted as spurious. It is further shown that the writer of the tablet, which has come down to us from remote antiquity, was himself an ignorant copyist of an earlier tablet or tablets; but of what date are the observations of this earlier tablet or tablets there is no evidence to show, except the antique style, and the fact of their belonging to the collection supposed to have been made by Sargon of Aganè; which tends to refer them to a period older than 1700 years B.C.

The length to which this comment has extended, and other circumstances, compel us to postpone the discussion of the identification of stars, &c.

Addition to a Paper entitled "On the Theoretical Value of the Acceleration of the Moon's Mean Motion in Longitude produced by the Change of Excentricity of the Earth's Orbit." By Sir G. B. Airy, K.C.B., Astronomer Royal.

In a paper lately communicated to the Society, my principal object was to exhibit the power of a new or factorial method of obtaining the disturbance of the Moon's movements depending on the introduction or change of an external force; and I applied it to the disturbance called the "Acceleration of the Moon's Mean Motion," produced by a gradual change in the excentricity of the earth's orbit. In completing the calculation, and estimating for that purpose the external factor of the formula, I limited myself to the imperfect expressions for the change of magnitude of the force, which (historically) had been adopted in the earliest investigations. I now propose to employ the more complete formula; referring, when advantageous, to the preceding paper, and using it to abbreviate this communication where it appears possible to do so.

(19.) The Sun's disturbing action is to be treated here in the same form as in the preceding articles; employing the symbol T for the ecliptic tangential accelerating force, and P for the ecliptic radial force measured from the Earth, and using V and v for the true longitudes of the Sun and Moon, R for the Sun's radius vector, and r for the Moon's radius vector, A for

the semi-axis major of the Sun's apparent orbit, and a for the reciprocal of the non-periodic term in the reciprocal of the Moon's radius vector. The units of measure have been explained in the preceding part of the paper. Then—

$$T \cdot \frac{1}{a} \cdot \frac{r}{a} = -\cdot 0083928^{*} \cdot \left(\frac{A}{R}\right)^{3} \cdot \left(\frac{r}{a}\right)^{2} \cdot \sin \lceil 2(v-V) \rceil + \cdot 0000053 \left(\frac{A}{R}\right)^{4} \cdot \left(\frac{r}{a}\right)^{3} \cdot \sin \lceil v-V \rceil ;$$

$$P \cdot \frac{1}{a} \cdot \frac{r}{a} = +\cdot 0083928^{*} \cdot \left(\frac{A}{R}\right)^{3} \cdot \left(\frac{r}{a}\right)^{2} \cdot \left\{ +\frac{1}{3} + \cos \lceil 2(v-V) \rceil \right\} - \cdot 0000160 \cdot \left(\frac{A}{R}\right)^{4} \cdot \left(\frac{r}{a}\right)^{3} \cdot \cos \lceil v-V \rceil ;$$

with some very small terms depending on the argument $\lceil 3(v-V) \rceil$. These latter terms, as will be seen below, can only produce available results by being multiplied by other very small terms of nearly the same form, and the results will be far below our limits of accuracy for other terms. As regards $\cos \lceil v-V \rceil$ in P (above), its factor is less than $\frac{1}{300}$ of the factor of $\cos \lceil 2(v-V) \rceil$, and it will be multiplied by terms whose coefficients are less than $\frac{1}{10}$ of those of $\cos \lceil 2(v-V) \rceil$, so that its efficient result will be less than $\frac{1}{3000}$ part. And that for T will be less than $\frac{1}{15000}$ part. I therefore reject these terms, and use only the following:

$$T \cdot \frac{1}{a} \cdot \frac{r}{a} = -\cdot 0083928 \times \left(\frac{A}{R}\right)^{3} \cdot \left(\frac{r}{a}\right)^{2} \times \sin \lceil 2(v-V) \rceil ,$$

$$P \cdot \frac{1}{a} \cdot \frac{r}{a} = +\cdot 0083928 \times \left(\frac{A}{R}\right)^{3} \cdot \left(\frac{r}{a}\right)^{2} \times \left\{ +\frac{1}{3} + \cos \lceil 2(v-V) \rceil \right\} .$$

(20.) Before entering on the details of treatment of these formulæ, it will be convenient to premise the following remarks.

Our object is, to ascertain the effect produced in our numerical expressions, by a numerical change δE of the numerical quantity E, the excentricity of the solar orbit. It is necessary therefore to know what power of E has been employed in forming our numerical expressions. Now, E is always accompanied, in the first steps of expansion, by an argument containing S. It is true that E^3 or E^5 may have contributed in some instances (not in general) to form a portion of the coefficient of that argument containing S; but, if so, that portion of the coefficient must have received the numerical multiplier E^2 or E^4, either of which is extremely small, and which will have affected

* By a fault of transcription, erroneous numbers are printed in the first investigation, p. 371. The term $-\cdot 0077$ is not wanted here, being included in the further investigations.

a very small part of the coefficient. Thus we may assume, without perceptible error, that the numerical coefficient of

$$\cos\lceil S\rceil, \quad \text{or} \quad \cos\lceil D-S\rceil, \ \&c.,$$

contains in itself the factor E; that the coefficient of

$$\cos\lceil 2S\rceil, \quad \text{or} \quad \cos\lceil 2D-2S\rceil, \quad \text{or} \quad \sin\lceil 2D-2S\rceil, \ \&c.,$$

contains in itself the factor E^2, and so on. (This, it will be observed, applies to the developments of R and V, but not always to more complicated functions, in which the trace of S may be destroyed by the union of two arguments; in these, the terms must be multiplied with due attention to their formation.) And, the power of E being thus determined, the variation of the coefficient, depending on δE, will be conveniently expressed thus. Let Q be a numerical coefficient containing the n^{th} power of E, so that $Q=W.E^n$ (W being a numeral, or a trigonometrical function, &c.). Then

$$\delta Q = n.W.E^{n-1}.\delta E = n.W.E^n.\frac{\delta E}{E} = n.Q.\frac{\delta E}{E}.$$

(21.) We shall now proceed to form the numerical value of $\left(\frac{A}{R}\right)^3.\left(\frac{r}{a}\right)^2$, with its variation depending on δE.

First, to form $\left(\frac{A}{R}\right)^3$,

Le Verrier gives for $\frac{R}{A}$,

$$+1{\cdot}0000000 \ \overset{E^2}{+{\cdot}0001406} \ \overset{E}{-{\cdot}0167687}\cdot\cos\lceil S\rceil \ \overset{E^2}{-{\cdot}0001406}\cdot\cos\lceil 2S\rceil \ \overset{E^3}{-{\cdot}0000018}\cdot\cos\lceil 3S\rceil;$$

(in which the symbols E, E^2, E^3, placed above the figures, indicate the power of E from which those numbers have been formed): and from this, by ordinary expansion, we find for $\left(\frac{A}{R}\right)^3$,

$$+\left\{\begin{matrix}+1.0000000 & \overset{E^2}{+{\cdot}0004215}\end{matrix}\right\}+\left\{\begin{matrix}\overset{E}{+{\cdot}0503061} & \overset{E^3}{+{\cdot}0000114}\end{matrix}\right\}\cos\lceil S\rceil$$

$$\overset{E^2}{+{\cdot}0012651}\cdot\cos\lceil 2S\rceil \ \overset{E^3}{+0000309}\cdot\cos\lceil 3S\rceil;$$

and, forming the variation on the principles just described, and attaching it to the principal terms, we obtain for $\left(\frac{A}{R}\right)^3 + \delta \cdot \left(\frac{A}{R}\right)^3$,

$$+ 1{\cdot}0004215 + {\cdot}0503275 \cdot \cos\lceil \overline{S} \rceil + {\cdot}0012651 \cdot \cos\lceil \overline{2S} \rceil + {\cdot}0000309 \cdot \cos\lceil \overline{3S} \rceil$$

$$+ \frac{\delta E}{E} \times \Big\{ + {\cdot}0008430 + {\cdot}0503703 \cdot \cos\lceil \overline{S} \rceil + {\cdot}0025302 \cdot \cos\lceil \overline{2S} \rceil + {\cdot}0000927 \cdot \cos\lceil \overline{3S} \rceil \Big\}.$$

Secondly, to form $\left(\frac{r}{a}\right)^2$.

The expression which I have found by expansion from Delaunay's coefficients, including no arguments excepting those which contain D, S, and their combinations, is

$$+ 1{\cdot}0046872 + {\cdot}0005426 \cdot \cos\lceil D \rceil - {\cdot}0148320 \cdot \cos\lceil 2D \rceil + {\cdot}0002763 \cdot E \cos\lceil S \rceil$$

$$+ {\cdot}0000075 \cdot E^2 \cos\lceil 2S \rceil - {\cdot}0000880 \cdot E \cos\lceil D+S \rceil + {\cdot}0000021 \cdot E \cos\lceil D-S \rceil$$

$$+ {\cdot}0001624 \cdot E \cos\lceil 2D+S \rceil - {\cdot}0010217 \cdot E \cos\lceil 2D-S \rceil - {\cdot}0000421 \cdot E^2 \cos\lceil 2D-2S$$

$$- {\cdot}0000021 \cdot E \cos\lceil 4D+S \rceil + {\cdot}0000082 \cdot E \cos\lceil 4D-S \rceil,$$

which gives $\left(\frac{r}{a}\right)^2$, requiring no further change.

Then for $\delta \cdot \left(\frac{r}{a}\right)^2$

we have,

$$+ \frac{\delta E}{E} \times \Big\{ + {\cdot}0002763 \cdot \cos\lceil \overline{S} \rceil + {\cdot}0000150 \cdot \cos\lceil \overline{2S} \rceil - {\cdot}0000880 \cdot \cos\lceil \overline{D+S} \rceil$$
$$+ {\cdot}0000021 \cdot \cos\lceil \overline{D-S} \rceil + {\cdot}0001624 \cdot \cos\lceil \overline{2D+S} \rceil$$
$$- {\cdot}0010217 \cdot \cos\lceil \overline{2D-S} \rceil - {\cdot}0000842 \cdot \cos\lceil \overline{2D-2S}$$
$$- {\cdot}0000021 \cdot \cos\lceil \overline{4D+S} \rceil + {\cdot}0000082 \cdot \cos\lceil \overline{4D-S} \rceil \Big\}.$$

Multiplying the expression for

$$\left(\frac{A}{R}\right)^3 + \delta \cdot \left(\frac{A}{R}\right)^3$$

by that for

$$\left(\frac{r}{a}\right)^2 + \delta \cdot \left(\frac{r}{a}\right)^2$$

we obtain the expression for

$$\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 + \delta \cdot \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2,$$

as follows :—

Value of $\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2$.	Factors of $\frac{\delta E}{E}$ in the Variation of $\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2$. (The first line in each bracket gives the factor depending on $\left(\frac{A}{R}\right)^3$; the second gives that depending on $\left(\frac{r}{a}\right)^2$).	
+ 1·0051177	{ + ·0008539 / + ·0000070 }	= + ·0008609
+ ·0005407 . cos |D|	{ − ·0000017 / − ·0000021 }	= − ·0000038 . cos |D|
− ·0148598 . cos |2D|	{ − ·0000342 / − ·0000217 }	= − ·0000559 . cos |2D|
+ ·0000001 . cos |4D|	{ + ·0000001 / + ·0000001 }	= + ·0000002 . cos |4D|
+ ·0508402 . cos |S|	{ + ·0506071 / + ·0002770 }	= + ·0508841 . cos |S|
+ ·0012855 . cos |2S|	{ + ·0025490 / + ·0000220 }	= + ·0025710 . cos |2S|
+ ·0000314 . cos |3S|	{ + ·0000936 / + ·0000006 }	= + ·0000942 . cos |3S|
− ·0000743 . cos |D+S|	{ + ·0000136 / − ·0000880 }	= − ·0000744 . cos |D+S|
+ ·0000158 . cos |D−S|	{ + ·0000135 / + ·0000020 }	= + ·0000155 . cos |D−S|
− ·0000019 . cos |D+2S|	{ − ·0000015 / − ·0000022 }	= − ·0000037 . cos |D+2S|
+ ·0000004 . cos |D−2S|	{ + ·0000008 / + ·0000001 }	= + ·0000009 . cos |D−2S|
+ ·0000000 . cos |D+3S|	{ − ·0000001 / − ·0000001 }	= − ·0000002 . cos |D+3S|
− ·0002114 . cos |2D+S|	{ − ·0003747 / + ·0001619 }	= − ·0002128 . cos |2D+S|
− ·0013963 . cos |2D−S|	{ − ·0003752 / − ·0010241 }	= − ·0013993 . cos |2D−S|
− ·0000053 . cos |2D+2S|	{ − ·0000147 / + ·0000041 }	= − ·0000106 . cos |2D+2S|

Value of $\left(\frac{A}{R}\right)^3 . \left(\frac{r}{a}\right)^2$.	Factors of $\frac{\delta E}{E}$ in the Variation of $\left(\frac{A}{R}\right)^3 . \left(\frac{r}{a}\right)^2$. (The first line in each bracket gives the factor depending on $\left(\frac{A}{R}\right)^3$; the second gives that depending on $\left(\frac{r}{a}\right)^2$).	
$- \cdot0000772 . \cos \lceil 2D-2S \rceil$	$\left\{\begin{matrix} -\cdot0000445 \\ -\cdot0001099 \end{matrix}\right\}$	$= -\cdot0001544 . \cos \lceil 2D-2S \rceil$
$- \cdot0000001 . \cos \lceil 2D+3S \rceil$	$\left\{\begin{matrix} -\cdot0000005 \\ +\cdot0000001 \end{matrix}\right\}$	$= -\cdot0000004 . \cos \lceil 2D+3S \rceil$
$- \cdot0000020 . \cos \lceil 2D-3S \rceil$	$\left\{\begin{matrix} -\cdot0000031 \\ -\cdot0000027 \end{matrix}\right\}$	$= -\cdot0000058 . \cos \lceil 2D-3S \rceil$
$\cdot0000000 . \cos \lceil 2D-4S \rceil$	$\left\{\begin{matrix} -\cdot0000001 \\ -\cdot0000001 \end{matrix}\right\}$	$= -\cdot0000002 . \cos \lceil 2D-4S \rceil$
$- \cdot0000021 . \cos \lceil 4D+S \rceil$	$\left\{\begin{matrix} \cdot0000000 \\ -\cdot0000021 \end{matrix}\right\}$	$= -\cdot0000021 . \cos \lceil 4D+S \rceil$
$+ \cdot0000082 . \cos \lceil 4D-S \rceil$	$\left\{\begin{matrix} \cdot0000000 \\ +\cdot0000082 \end{matrix}\right\}$	$= +\cdot0000082 . \cos \lceil 4D-S \rceil$
$+ \cdot0000002 . \cos \lceil 4D-2S \rceil$	$\left\{\begin{matrix} +\cdot0000002 \\ +\cdot0000002 \end{matrix}\right\}$	$= +\cdot0000004 . \cos \lceil 4D-2S \rceil$
$- \cdot0000001 . \cos \lceil 4D+2S \rceil$	$\left\{\begin{matrix} -\cdot0000001 \\ -\cdot0000001 \end{matrix}\right\}$	$= -\cdot0000002 . \cos \lceil 4D+2S \rceil$

Several of the numbers given above can produce no effect in our ultimate result; but it is less difficult and more satisfactory to include all than to make a selection.

(22.) We now proceed with the formation of the quantities which multiply $\left(\frac{A}{R}\right)^3 . \left(\frac{r}{a}\right)^2$; commencing with the argument $\lceil 2(v-V) \rceil$.

Referring again to Delaunay and Leverrier, and putting nt and Nt for the mean longitudes of the Moon and Sun,

$$\begin{aligned} v = nt &- \cdot0006169 . \sin \lceil D \rceil + \cdot0114887 . \sin \lceil 2D \rceil \\ &- \cdot0032422 . \sin \lceil S \rceil - \cdot0000363 . \sin \lceil 2S \rceil \\ &- \cdot0000007 . \sin \lceil 3S \rceil - \cdot0001192 . \sin \lceil 2D+S \rceil \\ &+ \cdot0008016 . \sin \lceil 2D-S \rceil : \end{aligned}$$

$$V = Nt + 0335409 . \sin \lceil S \rceil + \cdot0003515 . \sin \lceil 2S \rceil + \cdot0000050 . \sin \lceil 3S \rceil .$$

Therefore,

$$\begin{aligned} v-V = \lceil nt-Nt \rceil &- \cdot0006169 . \sin \lceil D \rceil + \cdot0114887 . \sin \lceil 2D \rceil \\ &- \cdot0367831 . \sin \lceil S \rceil - \cdot0003878 . \sin \lceil 2S \rceil - \cdot0000057 . \sin \lceil 3S \rceil \\ &- \cdot0001192 . \sin \lceil 2D+S \rceil + \cdot0008016 . \sin \lceil 2D-S \rceil . \end{aligned}$$

But $\overline{|nt-Nt|} = D$. Making this substitution, and doubling the whole;

$$\overline{2(v-V)} = \left\{ \begin{array}{l} 2D \\ \quad +(-\cdot0012338 \cdot \sin\overline{D} + \cdot0229774 \cdot \sin\overline{2D} \\ \quad -\cdot0735662 \cdot \sin\overline{S} - \cdot0007756 \cdot \sin\overline{2S} - \cdot0000114 \cdot \sin\overline{3S} \\ \quad -\cdot0002384 \cdot \sin\overline{2D+S} + \cdot0016032 \cdot \sin\overline{2D-S}) \end{array} \right\}$$

We shall call this

$$\overline{2D} + \Delta(2D).$$

Then

$$\sin\overline{2(v-V)} = \sin\overline{2D} \cdot \cos\overline{\Delta(2D)} + \cos\overline{2D} \cdot \sin\overline{\Delta(2D)};$$
$$\cos\overline{2(v-V)} = \cos\overline{2D} \cdot \cos\overline{\Delta(2D)} - \sin\overline{2D} \cdot \sin\overline{\Delta(2D)};$$

and we must now ascertain the values of $\cos\overline{\Delta(2D)}$ and $\sin\overline{\Delta(2D)}$.

(23.) The series giving these values are

$$\cos\overline{\Delta(2D)} = 1 - \frac{1}{2}\left\{\Delta(2D)\right\}^2 + \frac{1}{24}\left\{\Delta(2D)\right\}^4 - \&c.$$
$$\sin\overline{\Delta(2D)} = \Delta(2D) - \frac{1}{6}\left\{\Delta(2D)\right\}^3 + \frac{1}{120}\left\{\Delta(2D)\right\}^5 - \&c.$$

Now, it appears on trial of the principal terms, with due regard of the repeated bisections in combinations of arguments, that $\frac{1}{24}\{\Delta(2D)\}^4$ and higher powers are not sensible for the investigation before us. Therefore we have

$$\sin\overline{2(v-V)} = \sin\overline{2D} \cdot \left\{1 - \frac{1}{2}\{\Delta(2D)\}^2\right\} + \cos\overline{2D} \cdot \left\{\Delta(2D) - \frac{1}{6}\{\Delta(2D)\}^3\right\};$$
$$\cos\overline{2(v-V)} = \cos\overline{2D} \cdot \left\{1 - \frac{1}{2}\{\Delta(2D)\}^2\right\} - \sin\overline{2D} \cdot \left\{\Delta(2D) - \frac{1}{6}\{\Delta(2D)\}^3\right\};$$

each of which is ultimately to be multiplied by

$$\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2.$$

(24.) For $\{\Delta(2D)\}^2$, I have thought it desirable to form every product as low as ·0000001 (rejecting a few of the smallest); but for $\{\Delta(2D)\}^3$, I have been satisfied with multiplying the four largest terms of $\{\Delta(2D)\}^2$ by the two largest terms of $\Delta(2D)$. Thus we obtain:—

$\Delta(2D)$		$\{\Delta(2D)\}^2$		$\{(\Delta 2D)\}^3$	
		Non-periodic $= (+\cdot0029723 + \cdot0054158 \times \frac{\delta E}{E})$			
$+\sin\lceil D\rceil$	$\times(-\cdot0012338$	$+\cos\lceil D\rceil \times(-\cdot0000284$	)		
$+\sin\lceil 2D\rceil$	$\times(+\cdot0229774$	$+\cos\lceil 2D\rceil \times(+\cdot001349 + \cdot0002712 \times \frac{\delta E}{E})$		$+\sin\lceil 2D\rceil \times(+\cdot0001926 + \cdot0003716 \times \frac{\delta E}{E})$	
		$+\cos\lceil 3D\rceil \times(+\cdot0000284$	)		
		$+\cos\lceil 4D\rceil \times(-\cdot0002636 + \cdot0000008 \times \frac{\delta E}{E})$			
$+\sin\lceil S\rceil$	$\times(-\cdot0735662 - \cdot0735662 \times \frac{\delta E}{E})$	$+\cos\lceil S\rceil \times(+\cdot0000883 + \cdot0002023 \times \frac{\delta E}{E})$		$+\sin\lceil S\rceil \times(-\cdot0003753 - \cdot0009502 \times \frac{\delta E}{E})$	
$+\sin\lceil 2S\rceil$	$+(\cdot\cdot0007756 - \cdot0015512 \times \frac{\delta E}{E})$	$+\cos\lceil 2S\rceil \times(-\cdot0027056 - \cdot0054096 \times \frac{\delta E}{E})$			
$+\sin\lceil 3S\rceil$	$\times(-\cdot0000114 - \cdot0000342 \times \frac{\delta E}{E})$	$+\cos\lceil 3S\rceil \times(-\cdot0000570 - \cdot0001710 \times \frac{\delta E}{E})$		$+\sin\lceil 3S\rceil \times(+\cdot0000996 + \cdot0002988 \times \frac{\delta E}{E})$	
		$+\cos\lceil 4S\rceil \times(-\cdot0000011 - \cdot0000044 \times \frac{\delta E}{E})$			
		$+\cos\lceil D+S\rceil \times(-\cdot0000906 - \cdot0000906 \times \frac{\delta E}{E})$			

(*Continued.*)

$\Delta(2D)$	$\{\Delta(2D)\}^2$	$\{\Delta(2D)\}^3$
	$+\cos\lvert D-S\rvert \times (+\cdot0000888 \quad +\cdot0000888\times\frac{\delta E}{E})$	
$+\sin\lvert 2D+S\rvert \times (-\cdot0002384 \quad -\cdot0002384\times\frac{\delta E}{E})$	$+\cos\lvert 2D+S\rvert \times (+\cdot0016915 \quad +\cdot0016939\times\frac{\delta E}{E})$	
$+\sin\lvert 2D-S\rvert \times (+\cdot0016032 \quad +\cdot0016032\times\frac{\delta E}{E})$	$+\cos\lvert 2D-S\rvert \times (-\cdot0016901 \quad -\cdot0016896\times\frac{\delta E}{E})$	
		$+\sin\lvert 2D+2S\rvert \times (-\cdot0000933 \quad -\cdot0001866\times\frac{\delta E}{E})$
	$+\cos\lvert 2D-2S\rvert \times (-\cdot0001358 \quad -\cdot0002716\times\frac{\delta E}{E})$	$+\sin\lvert 2D-2S\rvert \times (-\cdot0000933 \quad -\cdot0001866\times\frac{\delta E}{E})$
	$+\cos\lvert 2D-3S\rvert \times (-\cdot0000014 \quad -\cdot0000042\times\frac{\delta E}{E})$	
	$+\cos\lvert 3D-S\rvert \times (+\cdot0000020 \quad +\cdot0000020\times\frac{\delta E}{E})$	
	$+\cos\lvert 4D+S\rvert \times (+\cdot0000056 \quad +\cdot0000056\times\frac{\delta E}{E})$	$+\sin\lvert 4D+S\rvert \times (+\cdot0000195 \quad +\cdot0000195\times\frac{\delta E}{E})$
	$+\cos\lvert 4D-S\rvert \times (-\cdot0000369 \quad -\cdot0000369\times\frac{\delta E}{E})$	$+\sin\lvert 4D-S\rvert \times (-\cdot0000195 \quad -\cdot0000195\times\frac{\delta E}{E})$
	$+\cos\lvert 4D-2S\rvert \times (-\cdot0000013 \quad -\cdot0000026\times\frac{\delta E}{E})$	

(25.) From these we form the following expressions for $\cos\lceil\Delta(2D)\rceil$ and $\sin\lceil\Delta(2D)\rceil$, equal respectively to $1-\frac{1}{2}\{\Delta(2D)\}^2$ and $\Delta(2D)-\frac{1}{6}\{\Delta(2D)\}^3$.

$$\cos\lceil\Delta(2D)\rceil$$

$$\text{Non-periodic} = (+\ .9985138 \quad -\ .0027079 \times \frac{\delta E}{E})$$

$$\overline{+}\cos\lceil D\rceil \times (+\ .0000142 \quad)$$

$$+\cos\lceil 2D\rceil \times (-\ .0000674 \quad -\ .0001356 \times \frac{\delta E}{E})$$

$$+\cos\lceil 3D\rceil \times (-\ .0000142 \quad)$$

$$+\cos\lceil 4D\rceil \times (+\ .0001318 \quad -\ .0000004 \times \frac{\delta E}{E})$$

$$+\cos\lceil S\rceil \times (-\ .0000442 \quad -\ .0001012 \times \frac{\delta E}{E})$$

$$+\cos\lceil 2S\rceil \times (+\ .0013528 \quad +\ .0027048 \times \frac{\delta E}{E})$$

$$+\cos\lceil 3S\rceil \times (+\ .0000285 \quad +\ .0000855 \times \frac{\delta E}{E})$$

$$+\cos\lceil 4S\rceil \times (+\ .0000006 \quad +\ .0000022 \times \frac{\delta E}{E})$$

$$\sin\lceil\Delta(2D)\rceil$$

$$+\sin\lceil D\rceil \times (-\ .0012338$$

$$+\sin\lceil 2D\rceil \times (+\ .0229453 \quad -\ .0000619 \times \frac{\delta E}{E})$$

$$+\sin\lceil S\rceil \times (-\ .0735066 \quad -\ .734078 \times \frac{\delta E}{E})$$

$$+\sin\lceil 2S\rceil \times (-\ .0007756 \quad -\ .0015512 \times \frac{\delta E}{E})$$

$$+\sin\lceil 3S\rceil \times (-\ .0000280 \quad -\ .0000840 \times \frac{\delta E}{E})$$

(*Continued.*)

	$\cos\lvert\Delta(2D)\rvert$		$\sin\lvert\Delta(2D)\rvert$	
$+\cos\lvert D+S\rvert$	$\times\left(+\ .0000453\right.$	$\left.+\ .0000453\times\frac{\delta E}{E}\right)$		
$+\cos\lvert D-S\rvert$	$\times\left(-\ .0000444\right.$	$\left.-\ .0000444\times\frac{\delta E}{E}\right)$		
$+\cos\lvert 2D+S\rvert$	$\times\left(-\ .0008458\right.$	$\left.-\ .0008470\times\frac{\delta E}{E}\right)$	$+\sin\lvert 2D+S\rvert$	$\times\left(-\ .0002384\right.$ $\left.-\ .0002384\times\frac{\delta E}{E}\right)$
$+\cos\lvert 2D-S\rvert$	$\times\left(+\ .0008450\right.$	$\left.+\ .0008448\times\frac{\delta E}{E}\right)$	$+\sin\lvert 2D-S\rvert$	$\times\left(+\ .0016032\right.$ $\left.+\ .0016032\times\frac{\delta E}{E}\right)$
			$+\sin\lvert 2D+2S\rvert$	$\times\left(+\ .0000156\right.$ $\left.+\ .0000311\times\frac{\delta E}{E}\right)$
$+\cos\lvert 2D-2S\rvert$	$\times\left(+\ .0000679\right.$	$\left.+\ .0001358\times\frac{\delta E}{E}\right)$	$+\sin\lvert 2D-2S\rvert$	$\times\left(+\ .0000156\right.$ $\left.+\ .0000311\times\frac{\delta E}{E}\right)$
$+\cos\lvert 2D-3S\rvert$	$\times\left(+\ .0000007\right.$	$\left.+\ .0000021\times\frac{\delta E}{E}\right)$		
$+\cos\lvert 3D-S\rvert$	$\times\left(-\ .0000010\right.$	$\left.-\ .0000010\times\frac{\delta E}{E}\right)$		
$+\cos\lvert 4D+S\rvert$	$\times\left(-\ .0000028\right.$	$\left.-\ .0000028\times\frac{\delta E}{E}\right)$	$+\sin\lvert 4D+S\rvert$	$\times\left(-\ .0000033\right.$ $\left.-\ .0000033\times\frac{\delta E}{E}\right)$
$+\cos\lvert 4D-S\rvert$	$\times\left(+\ .0000184\right.$	$\left.+\ .0000184\times\frac{\delta E}{E}\right)$	$+\sin\lvert 4D-S\rvert$	$\times\left(+\ .0000033\right.$ $\left.+\ .0000033\times\frac{\delta E}{E}\right)$
$+\cos\lvert 4D-2S\rvert$	$\times\left(+\ .0000006\right.$	$\left.+\ .0000013\times\frac{\delta E}{E}\right)$		

(26.) We are now to combine the expressions last found, in order to produce the quantities

$$T \cdot \frac{1}{a} \cdot \frac{r}{a} \text{ and } P \cdot \frac{1}{a} \cdot \frac{r}{a},$$

or

$$-\cdot 0083928 \times \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \sin \lceil 2(v-V) \rceil,$$

$$+\cdot 0083928 \times \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \left\{ +\frac{1}{3} + \cos \lceil 2(v-V) \rceil \right\}$$

(of which we are to retain only the terms that multiply $\frac{\delta E}{E}$, or, as we may conveniently express it, $E'.t$, where E' is a constant), and to examine the results of substituting the combinations in the Factorial Equations. Our ultimate object is, not to retain every term in the solution of the equations, but only the non-periodic terms (increasing with the time) of the longitude and the radius vector, especially the longitude (which alone will ultimately possess any value for us). And this consideration is to be kept in view in solving the factorial equations. We will now consider separately the two disturbing functions.

(27.) On examining the formation of the selected terms of

$$\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \sin \lceil 2(v-V) \rceil,$$

or

$$\left(\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \cos \lceil \Delta(2D) \rceil \right) \times \sin \lceil 2D \rceil$$

$$+ \left(\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \sin \lceil \Delta(2D) \rceil \right) \times \cos \lceil 2D \rceil,$$

with reference to the expansions of these functions lately exhibited, it appears that the first part is the product of a constant (accompanied with a series of cosines) by a sine, and the second is the product of a series of sines by a cosine. Each of these products will be a series of sines. In order to ascertain whether these will produce any constant term in the ultimate result, it is necessary to refer to the factorial equations. Putting

$$Ft \cdot \sin \lceil Gt \rceil$$

for one of the terms of the expansion of T, considered alone (which is legitimate, because the equations are linear), we have the factorial equations,

$$\mathrm{F}t\,.\,\sin\lceil \mathrm{G}t\rceil = \left\{\begin{array}{ll} +\delta\left(\frac{a}{r}\right) & \times\{\ \ \cdot0000-\cdot0281\,.\,\sin\lceil 2\mathrm{D}\rceil\} \\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times\{-1\cdot9919+\cdot0061\,.\,\cos\lceil 2\mathrm{D}\rceil\} \\ +\delta v & \times\{\ \ \cdot0000+\cdot0168\,.\,\cos\lceil 2\mathrm{D}\rceil\} \\ +\frac{d}{dt}(\delta v) & \times\{\ \ \cdot0000+\cdot0268\,.\,\sin\lceil 2\mathrm{D}\rceil\} \\ +\frac{d^2}{dt^2}(\delta v) & \times\{+1\cdot0006-\cdot0144\,.\,\cos\lceil 2\mathrm{D}\rceil\} \end{array}\right\}.$$

$$0 = \left\{\begin{array}{ll} +\delta\left(\frac{a}{r}\right) & \times\{+2\cdot9964-\cdot0236\,.\,\cos\lceil 2\mathrm{D}\rceil\} \\ +\frac{d}{dt}\left(\delta\frac{a}{r}\right) & \times\{\ \ \cdot0000-\cdot0528\,.\,\sin\lceil 2\mathrm{D}\rceil\} \\ +\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right) & \times\{-1\cdot0053+\cdot0212\,.\,\cos\lceil 2\mathrm{D}\rceil\} \\ +\delta v & \times\{\ \ \cdot0000+\cdot0168\,.\,\sin\lceil 2\mathrm{D}\rceil\} \\ +\frac{d}{dt}(\delta v) & \times\{-1\cdot9888-\cdot0090\,.\,\cos\lceil 2\mathrm{D}\rceil\} \end{array}\right\}.$$

Using the symbols

$$\Phi = -\cdot0281\,.\,\delta\left(\frac{a}{r}\right).\sin\lceil 2\mathrm{D}\rceil + \cdot0061\,.\,\frac{d}{dt}\left(\delta\frac{a}{r}\right).\cos\lceil 2\mathrm{D}\rceil + \cdot0168\,.\,\delta v\,.\,\cos\lceil 2\mathrm{D}\rceil + \cdot0268\,.\,\frac{d}{dt}(\delta v)\,.\,\sin 2\mathrm{D} - \cdot0144\,.\,\frac{d^2}{dt^2}(\delta v)\,.\,\cos\lceil 2\mathrm{D}\rceil;$$

$$\Psi = -\cdot0236\,.\,\delta\left(\frac{a}{r}\right).\cos\lceil 2\mathrm{D}\rceil - \cdot0528\,.\,\frac{d}{dt}\left(\delta\frac{a}{r}\right).\sin\lceil 2\mathrm{D}\rceil + \cdot0212\,.\,\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right).\cos\lceil 2\mathrm{D}\rceil + \cdot0168\,.\,\delta v\,.\,\sin\lceil 2\mathrm{D}\rceil - \cdot0090\,.\,\frac{d}{dt}(\delta v).\cos\lceil 2\mathrm{D}\rceil$$

the equations may be conveniently written,

$$\begin{array}{lll} -1\cdot9919\,.\,\frac{d}{dt}\left(\delta\frac{a}{r}\right)+1\cdot0006\,.\,\frac{d^2}{dt^2}(\delta v) & = \mathrm{F}t\,.\,\sin\lceil \mathrm{G}t\rceil - \Phi; \\ +2\cdot9964\,.\,\delta\left(\frac{a}{r}\right)-1\cdot0053\,.\,\frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right)-1\cdot9888\,.\,\frac{d}{dt}(\delta v) & = 0 \qquad -\Psi \end{array}$$

(in which we shall afterwards use, instead of the fractional factors, the integers -2, $+1$, $+3$, -1, -2); and these equations will be most conveniently solved by successive substitution; first, neglecting the small quantities Φ and Ψ; then solving the equations so limited, substituting these limited solutions in Φ and Ψ; and then re-solving; and so on. It is easily found that limited solutions are

$$\delta\left(\frac{a}{r}\right) = \frac{2\mathrm{G}}{\mathrm{G}^2-\mathrm{G}^4}\,\mathrm{F}t\,.\,\cos\lceil \mathrm{G}t\rceil, \qquad \delta v = \frac{3+\mathrm{G}^2}{\mathrm{G}^2-\mathrm{G}^4}\,\mathrm{F}t\,.\,\sin\lceil \mathrm{G}t\rceil;$$

and these, on substitution, destroy the term whose factor is Ft, but introduce other terms. Upon making the substitutions

$$\delta\left(\frac{a}{r}\right) = \frac{2G}{G^2-G^4}\,\mathrm{F}t\,.\,\cos\lceil Gt\rceil + c, \qquad \delta v = \frac{3+G^2}{G^2-G^4}\,\mathrm{F}t\,.\,\sin\lceil Gt\rceil + s,$$

we obtain the following equations,

$$-2\,.\,\frac{dc}{dt} + \frac{d^2s}{dt^2} = -\frac{2G+2G^3}{G^2-G^4}\,\mathrm{F}\,.\,\cos\lceil Gt\rceil - \Phi;$$

$$+3.c - \frac{d^2c}{dt^2} - 2.\frac{ds}{dt} = +\frac{6-2G^2}{G^2-G^4}\,\mathrm{F}\,.\,\sin\lceil Gt\rceil - \Psi;$$

and, on substituting in Φ and Ψ the first terms of the expressions for $\delta\left(\frac{a}{r}\right)$, δv, and their differentials, we find equations exactly similar to those whose solution is given in Articles 11 and 12. The precise values of the coefficients for $\delta\frac{a}{r}$ and δv are of no importance; at present it is only necessary for us to remark that they are constants for each value of G.

We shall now consider the character of the expressions for $\delta\frac{a}{r}$ and δv: first, for the general value of G; secondly for the the special value $Gt = 2D$.

For the general case, we are to substitute in Φ and Ψ the very approximate values of $\delta\left(\frac{a}{r}\right)$ and δv given by the formulæ

$$\delta\left(\frac{a}{r}\right) = \mathrm{C}t\,.\,\cos\lceil Gt\rceil, \quad \delta v = \mathrm{S}t\,.\,\sin\lceil Gt\rceil;$$

and we find

$$\Phi = \left\{\begin{array}{ll} +\{+\cdot0281\,.\,\mathrm{C} - \cdot0268\,.\,\mathrm{GS}\} & \times t\,.\,\sin\lceil 2D\rceil\,.\,\cos\lceil Gt\rceil \\ +\{+\cdot0061\,.\,\mathrm{GC} - \cdot0144\,.\,\mathrm{G^2S} - \cdot0168\,.\,\mathrm{S}\} & \times t\,.\,\cos\lceil 2D\rceil\,.\,\sin\lceil Gt\rceil \\ +\{-\cdot0061\,.\,\mathrm{C} + \cdot0288\,.\,\mathrm{GS}\} & \times \cos\lceil 2D\rceil\,.\,\cos\lceil Gt\rceil \\ +\{-\cdot0268\,.\,\mathrm{S}\} & \times \sin\lceil 2D\rceil\,.\,\sin\lceil Gt\rceil \end{array}\right\}$$

$$\Psi = \left\{\begin{array}{ll} +\{+\cdot0236\,.\,\mathrm{C} + \cdot0212\,.\,\mathrm{G^2C} + \cdot0090\,.\,\mathrm{GS}\} & \times t\,.\,\cos\lceil 2D\rceil\,.\,\cos\lceil Gt\rceil \\ +\{-\cdot0528\,.\,\mathrm{GC} - \cdot0168\,.\,\mathrm{S}\} & \times t\,.\,\sin\lceil 2D\rceil\,.\,\sin\lceil Gt\rceil \\ +\{+\cdot0528\,.\,\mathrm{C}\} & \times \sin\lceil 2D\rceil\,.\,\cos\lceil Gt\rceil \\ +\{+\cdot0424\,.\,\mathrm{C} + \cdot0090\,.\,\mathrm{S}\} & \times \cos\lceil 2D\rceil\,.\,\sin\lceil Gt\rceil \end{array}\right\}$$

Now, all these terms, as well as the two new terms in the equation lately formed, are periodical; and the form of the equations for c and s is the same as that for $\delta\left(\frac{a}{r}\right)$ and δv; and

therefore, to this second step of approximation as well as in the first, the value of δv contains no non-periodical factor of t. And, as far as can be seen, the same remark applies to succeeding approximations.

For the case when $Gt = 2D$ (which in Article 9 and succeeding Articles is written F) we have only to remark that the equations before us are already solved in Articles 9 and 12. For, as the treatment of A, and of B, and of C, in those equations, is absolutely independent, we have merely to extract (for the effect of the term $-A \cdot bt \cdot \sin \lceil 2D \rceil$ in Article 9) the multiples of Ab in Article 12. And, referring to the terms in Article 11 of which g and l are the factors, it will be seen that those terms are periodical.

Therefore the term

$$-\cdot 0083928 \times \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \sin \lceil 2(v - V) \rceil$$

produces no non-periodical term in δv.

(28.) On examining the formation of the terms of

$$\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \cos \lceil 2(v - V) \rceil,$$

it will be seen that they consist of two parts. One is the series of constants produced by the products of similar cosines or similar sines, to be attached to a term of the same form in

$$+\cdot 0083928 \times \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \frac{1}{3},$$

of which we shall treat shortly. The other part is a series of products of cosines or sines of different arguments, producing cosines of new arguments.

For the general term $\cos \lceil Gt \rceil$, in the part just mentioned, the same reasoning will apply as that which has been employed for $\sin \lceil Gt \rceil$; the expressions for Φ and Ψ being the same as those which are used in discussing the result of $\sin \lceil Gt \rceil$; and the same conclusion following—namely, that there is no non-periodic term.

In the special case when $Gt = 2D$, the solution has already been effected in Articles 9 and 12, in treating the factor C. And the result is, that it produces no non-periodical term.

Therefore the term

$$+\cdot 0083928 \times \cos \lceil 2(v - V) \rceil$$

produces no non-periodic term, except from those non-periodic parts which we shall associate with the non-periodic parts of

$$+\cdot 0083928 \times \left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2 \times \frac{1}{3}.$$

(29.) For the non-periodic parts, whatever be their origin, we have again to remark that the solution is given in Articles 9 and 12; where the term B*b.t* of Article 9 produces, in Article 12, for $\delta\left(\frac{a}{r}\right)$ the term $-Bbt$, and for δv the term $-Bbt^2$.

(30.) The general result of these investigations is, that we are merely to extract, from the details of the two products

$$+\cdot0083928\times\left(\frac{A}{R}\right)^3\cdot\left(\frac{r}{a}\right)^2\times\frac{1}{3},$$

and

$$+\cdot0083928\times\left(\frac{A}{R}\right)^3\cdot\left(\frac{r}{a}\right)^2\times\cos\lceil 2(v-V)\rceil,$$

the factors of $\frac{\delta E}{E}$ or $E't$; and to use them in forming the Factorial Equations. To this extraction from details we shall now give our attention.

(31.) Referring to the expansion in Article 22, it will appear that we must consider these three terms:

$$+\frac{1}{3}\left(\frac{A}{R}\right)^3\cdot\left(\frac{r}{a}\right)^2,$$

$$+\left(\left(\frac{A}{R}\right)^3\cdot\left(\frac{r}{a}\right)^2\times\cos\lceil\Delta(2D)\rceil\right)\times\cos\lceil 2D\rceil,$$

$$-\left(\left(\frac{A}{R}\right)^3\cdot\left(\frac{r}{a}\right)^2\times\sin\lceil\Delta(2D)\rceil\right)\times\sin\lceil 2D\rceil.$$

The first part is merely $\frac{1}{3}$ of the development in Article 21, and consists of a constant and a series of cosines.

The second part consists of the product of a constant and a series of cosines by

$$\cos\lceil 2D\rceil,$$

The third part consists of the product of a series of sines by

$$\sin\lceil 2D\rceil,$$

Each of these parts will give rise to non-periodic terms.

(32.) For development of the first part, we have merely to take one-third of the non-periodic term in Article 21. Here, and in the development of the following parts, we shall retain only the multiples of $\frac{\delta E}{E}$. Thus we obtain for the first part:

$$+\frac{\delta E}{E}\times+\cdot0002870.$$

For the second and third parts we shall proceed to form the products to which allusion is made in the last sentences of Article 26. The only arguments in

$$\left(\frac{A}{R}\right)^3\cdot\left(\frac{r}{a}\right)^2$$

which can combine with those of

$$\cos\lceil\overline{\Delta(2D)}\rceil \quad \text{and} \quad \sin\lceil\overline{\Delta(2D)}\rceil$$

to form

$$\cos\lceil\overline{2D}\rceil \quad \text{or} \quad \sin\lceil\overline{2D}\rceil$$

are the following:

	Term of $\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2$.		Term of $\cos\lceil\overline{\Delta(2D)}\rceil$ or $\sin\lceil\overline{\Delta(2D)}\rceil$.
*	Non-periodic	will combine with	$\lceil\overline{2D}\rceil$
*	$\lceil\overline{D}\rceil$	,,	$\lceil\overline{D}\rceil$
	$\lceil\overline{D}\rceil$	,,	$\lceil\overline{3D}\rceil$
*	$\lceil\overline{2D}\rceil$	,,	non-periodic
*	$\lceil\overline{2D}\rceil$	,,	$\lceil\overline{4D}\rceil$
	$\lceil\overline{4D}\rceil$	,,	$\lceil\overline{2D}\rceil$
*	$\lceil\overline{S}\rceil$	,,	$\lceil\overline{2D+S}\rceil$
*	$\lceil\overline{S}\rceil$	,,	$\lceil\overline{2D-S}\rceil$
	$\lceil\overline{2S}\rceil$	,,	$\lceil\overline{2D+2S}\rceil$
*	$\lceil\overline{2S}\rceil$	,,	$\lceil\overline{2D-2S}\rceil$
	$\lceil\overline{3S}\rceil$	,,	$\lceil\overline{2D-3S}\rceil$
	$\lceil\overline{D+S}\rceil$	,,	$\lceil\overline{D-S}\rceil$
	$\lceil\overline{D-S}\rceil$	,,	$\lceil\overline{D+S}\rceil$
	$\lceil\overline{D-S}\rceil$	,,	$\lceil\overline{3D-S}\rceil$
*	$\lceil\overline{2D-S}\rceil$	,,	$\lceil\overline{S}\rceil$
	$\lceil\overline{2D-S}\rceil$	,,	$\lceil\overline{4D-S}\rceil$
*	$\lceil\overline{2D-2S}\rceil$	,,	$\lceil\overline{2S}\rceil$
	$\lceil\overline{2D-2S}\rceil$	,,	$\lceil\overline{4D-2S}\rceil$

Only those combinations which are distinguished by an asterisk can produce numbers included in the first seven decimals.

(33.) We now exhibit the products. It will be remembered that (except where one factor is non-periodic) each multiplication for $\cos\lceil\overline{\Delta(2D)}\rceil$ produces two cosines, of which we shall retain only one, namely, $\cos\lceil\overline{2D}\rceil$, and that its coefficient is to be only one-half of the product of the coefficients which form it. In like manner each multiplication for $\sin\lceil\overline{\Delta(2D)}\rceil$ produces two sines, of which we retain only $\sin\lceil\overline{2D}\rceil$.

Also, that we retain only the factors of $\frac{\delta E}{E}$.

The following are the special multiplications:—

PRODUCTS giving the factors of $\cos\lceil 2\overline{D}\rceil \cdot \frac{\delta E}{E}$, to be subsequently multiplied by $+\cos\lceil 2\overline{D}\rceil$.

Term of $\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2$			Term of $\cos\lceil\Delta(2\overline{D})\rceil$			Product multiplying $\cos\lceil 2\overline{D}\rceil \cdot \frac{\delta E}{E}$
Non-periodic	$(+1{\cdot}0051177$	$+{\cdot}0008609 \times \frac{\delta E}{E}) \times$	$+\cos\lceil 2\overline{D}\rceil$	$\times(-{\cdot}0000674$	$-{\cdot}0001356 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} -{\cdot}0001363 \\ 0000000 \end{matrix}\right.$
$+\cos\lceil 2\overline{D}\rceil$	$\times(-{\cdot}0148598$	$-{\cdot}0000559 \times \frac{\delta E}{E}) \times$	Non-periodic	$(+{\cdot}9985138$	$-{\cdot}0027079 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} +{\cdot}0000402 \\ -{\cdot}0000557 \end{matrix}\right.$
$+\cos\lceil S\rceil$	$\times(+{\cdot}0508402$	$+{\cdot}0508841 \times \frac{\delta E}{E}) \times$	$+\cos\lceil 2\overline{D}+S\rceil$	$\times(-{\cdot}0008458$	$-{\cdot}0008470 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} -{\cdot}0000431 \\ -{\cdot}0000430 \end{matrix}\right.$
$+\cos\lceil \overline{S}\rceil$	$\times(+{\cdot}0508402$	$+{\cdot}0508841 \times \frac{\delta E}{E}) \times$	$+\cos\lceil 2\overline{D}-S\rceil$	$\times(+{\cdot}0008450$	$+{\cdot}0008448 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} +{\cdot}0000429 \\ +{\cdot}0000430 \end{matrix}\right.$
$+\cos\lceil 2\overline{S}\rceil$	$\times(+{\cdot}0012855$	$+{\cdot}0025710 \times \frac{\delta E}{E}) \times$	$+\cos\lceil 2\overline{D}-2S\rceil$	$\times(+{\cdot}0000679$	$+{\cdot}0001358 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} +{\cdot}0000001 \\ +{\cdot}0000002 \end{matrix}\right.$
$+\cos\lceil 2\overline{D}-S\rceil$	$\times(-{\cdot}0013963$	$-{\cdot}0013993 \times \frac{\delta E}{E}) \times$	$+\cos\lceil S\rceil$	$\times(-{\cdot}0000442$	$-{\cdot}0001012 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} +{\cdot}0000001 \\ +{\cdot}0000001 \end{matrix}\right.$
$+\cos\lceil 2\overline{D}-2S\rceil$	$\times(-{\cdot}0000772$	$-{\cdot}0001544 \times \frac{\delta E}{E}) \times$	$+\cos\lceil 2\overline{S}\rceil$	$\times(+{\cdot}0013528$	$+{\cdot}0027048 \times \frac{\delta E}{E})$	$\left\{\begin{matrix} -{\cdot}0000002 \\ -{\cdot}0000002 \end{matrix}\right.$

All products, except those for the two first lines, are to be divided by 2.

PRODUCTS giving the factors of $\sin\lceil 2D\rceil \cdot \frac{\delta E}{E}$, to be subsequently multiplied by $-\sin\lceil 2D\rceil$.

	Term of $\left(\frac{A}{R}\right)^3 \cdot \left(\frac{r}{a}\right)^2$			Term of $\sin\lceil\Delta(2D)\rceil$			Product multiplying $\sin\lceil 2D\rceil \cdot \frac{\delta E}{E}$
Non-periodic	$(+1{\cdot}0051177$	$+{\cdot}0008609 \times \frac{\delta E}{E})\times$		$+\sin\lceil 2D\rceil$	$\times(+{\cdot}0229453$	$-{\cdot}0000619 \times \frac{\delta E}{E})$	$\left\{\begin{matrix}-{\cdot}0000622\\+{\cdot}0000197\end{matrix}\right\}$
$+\cos\lceil S\rceil$	$\times(+{\cdot}0508402$	$+{\cdot}0508841 \times \frac{\delta E}{E})\times$		$+\sin\lceil 2D+S\rceil$	$\times(-{\cdot}0002384$	$-{\cdot}0002384 \times \frac{\delta E}{E})$	$\left\{\begin{matrix}-{\cdot}0000121\\-{\cdot}0000121\end{matrix}\right\}$
$+\cos\lceil S\rceil$	$\times(+{\cdot}0508402$	$+{\cdot}0508841 \times \frac{\delta E}{E})\times$		$+\sin\lceil 2D-S\rceil$	$\times(+{\cdot}0016032$	$+{\cdot}0016032 \times \frac{\delta E}{E})$	$\left\{\begin{matrix}+{\cdot}0000815\\+{\cdot}0000816\end{matrix}\right\}$
$+\cos\lceil 2D-S\rceil$	$\times(-{\cdot}0013963$	$-{\cdot}0013993 \times \frac{\delta E}{E})\times$		$+\sin\lceil S\rceil$	$\times(-{\cdot}0735066$	$-{\cdot}0734078 \times \frac{\delta E}{E})$	$\left\{\begin{matrix}+{\cdot}0001025\\+{\cdot}0001029\end{matrix}\right\}$
$+\cos\lceil 2D-2S\rceil$	$\times(-{\cdot}0000772$	$-{\cdot}0001544 \times \frac{\delta E}{E})\times$		$+\sin\lceil 2S\rceil$	$\times(-{\cdot}0007756$	$-{\cdot}0015512 \times \frac{\delta E}{E})$	$\left\{\begin{matrix}+{\cdot}0000001\\+{\cdot}0000001\end{matrix}\right\}$

All products, except that for the first line, are to be divided by 2.

(34) The non-periodic terms in the three different parts of the effective perturbation-term

$$\text{P, or } +\cdot0083928\times\left(\frac{\text{A}}{\text{R}}\right)^3\cdot\left(\frac{r}{a}\right)^2\times\left\{\frac{1}{3}+\cos\left\lceil\overline{2(v-\text{V})}\right\rceil\right\}$$

may now be collected; and the numerical term to be inserted in the Factorial Equations may be formed.

The first part, or

$$+\frac{1}{3}\times\cdot0083928\times\left(\frac{\text{A}}{\text{R}}\right)^3\cdot\left(\frac{r}{a}\right)^2,$$

gives

$$+\cdot0083928\times+\cdot0002870\times\frac{\delta\text{E}}{\text{E}}.$$

For the second part, the summation of terms in the last table but one, performed in accordance with the rule at the bottom of the table, gives

$$-\cdot0001518\times\cos\lceil\overline{2\text{D}}\rceil\times\frac{\delta\text{E}}{\text{E}}.$$

This is to be multiplied by

$$+\cos\lceil\overline{2\text{D}}\rceil,$$

which gives for its non-periodic part

$$-\cdot0000759\times\frac{\delta\text{E}}{\text{E}}.$$

And the second part becomes

$$+\cdot0083928\times-\cdot0000759\times\frac{\delta\text{E}}{\text{E}}.$$

For the third part, the summation of terms in the last table, performed in like manner, gives

$$+\cdot0001298\times\sin\lceil\overline{2\text{D}}\rceil\times\frac{\delta\text{E}}{\text{E}}.$$

This is to be multiplied by

$$-\sin\lceil\overline{2\text{D}}\rceil,$$

producing the non-periodic term

$$-\cdot0000649\times\frac{\delta\text{E}}{\text{E}}.$$

And the third part becomes

$$+\cdot0083928 \times -\cdot0000649 \times \frac{\delta E}{E}.$$

The total value of P is thus

$$+\cdot0083928 \times +\cdot0001462 \times \frac{\delta E}{E}.$$

Now, by Le Verrier's Elements,

$$\frac{\text{variation of E for one year}}{E} = -\frac{\cdot000\,000\,4338}{\cdot016\,769\,27}.$$

And, as one year = 84 units of time (Article 3),

$$\frac{\delta E}{E} = -\frac{1}{84} \times \frac{\cdot000\,000\,4338}{\cdot016\,769\,27} \times t.$$

And therefore the total value of P is

$$-\cdot0083928 \times \cdot0001462 \times \frac{1}{84} \times \frac{\cdot000\,000\,4338}{\cdot016\,769\,27} \times t.$$

For the moment, we shall call this quantity $+Kt$.

(35.) An inspection of the operations in the former part of this paper (Articles 9 to 12) will show that the non-periodical part of the solution of the Factorial Equations is in no way affected by the periodical terms of P and T; and also that the periodical parts there retained in the expressions of the factors are unnecessary. And the use of integral numbers, instead of the fractional numbers in the coefficients entering into those factors, produces only unimportant error in the final result. With these simplifications, our equations become

$$\text{T or } \quad 0 = -2 \cdot \frac{d}{dt}\left(\delta\frac{a}{r}\right) + \frac{d^2}{dt^2}(\delta v).$$

$$\text{P or } +Kt = +3 \cdot \delta\left(\frac{a}{r}\right) - \frac{d^2}{dt^2}\left(\delta\frac{a}{r}\right) - 2 \cdot \frac{d}{dt}(\delta v).$$

It will easily be seen that the assumptions

$$\delta\frac{a}{r} = +Ct, \quad \delta v = +Ht^2,$$

are proper for the solution. Substituting

$$0 = -2C + 2H; \quad Kt = +3 \cdot Ct - 4 \cdot Ht;$$

from which

$$H = -K, \quad C = -K;$$

and the value of Ht^2, which is the 'Acceleration' that we are seeking, is $-Kt^2$. To compute the value corresponding to one year, we must make $t = 84$. The value of 'Acceleration' thus found, like all other angular measures here treated, is expressed in terms of radius; to reduce it to seconds, we must divide it by the length of 1″ in terms of radius = ·000004848137. Thus finally we obtain for Acceleration in longitude for one year, expressed in seconds of arc,

$$+ \cdot 0083928 \times \cdot 0001462 \times 84 \times \frac{\cdot 00000004338}{\cdot 01676927} \times \frac{1}{\cdot 000004848137};$$

$$= +0'' \cdot 00054997:$$

and for Acceleration in longitude for 100 years,

$$+5'' \cdot 4997.$$

(36.) The factor $\cos^2 \boxed{1}$, which enters into the original expressions for perturbing forces, has not hitherto been taken into account. Its mean value is ·99597; the Acceleration for one year is +0″·00054773; and finally,

The Acceleration in longitude for 100 years = 5″·4773.

I commenced this investigation without intention of mentioning in it any name. But I cannot terminate it without offering to Professor Adams my very hearty congratulations on his success in making a correction so large to a theory so important.

I think that the Lunar Theory is now placed in a difficult position. With the elements formerly received, the ancient eclipses were very well explained. With the modified theory, the agreement cannot be so good, and perhaps is impossible.

I am unwilling to abandon the interpretation of the ancient eclipses, and I think that reconciliation must be sought, in some new secular term, or in some alteration of the mean motions either of the Sun or of the Moon.

I apologise to the Society for two errors in the first part of the paper. Article 16 is totally erroneous, and Article 17 contains a numerical error. Both arose from the same circumstance, the haste in which the paper was closed for the Meeting of the Society.

1880, *July* 19.

On the Method of Least Squares. By J. W. L. Glaisher, M.A., F.R.S.

The present paper contains certain notes on the method of least squares; as they are to some extent connected with my previous paper "On the Solution of the Equations in the Method of Least Squares" (*Monthly Notices*, vol. xxxiv. pp. 311–334), the sections are numbered as if this were a continuation of that paper.

§ 14. The results of the treatment of a system of linear equations by the method of least squares may be stated as follows:—

Let

$$\begin{aligned} a_1x + b_1y + c_1z \ldots + f_1t &= n_1, \\ a_2x + b_2y + c_2z \ldots + f_2t &= n_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ a_mx + b_my + c_mz \ldots + f_mt &= n_m, \end{aligned}$$

be the m equations of condition, $x, y, z, \ldots t$ being the μ unknown quantities, and $n_1, n_2, \ldots n_m$ the m quantities obtained by observation.* The latter are assumed to be equally good—*i.e.* they are assumed to be such that the mean error of each of them is equal to the mean error ϵ of a standard observation.

The normal equations giving $x_0, y_0, \ldots t_0$, the "most probable" values of $x, y, \ldots t$ are

$$\begin{aligned} (aa)x_0 + (ab)y_0 + (ac)z_0 \ldots + (af)t_0 &= (an), \\ (ba)x_0 + (bb)y_0 + (bc)z_0 \ldots + (bf)t_0 &= (bn), \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ (fa)x_0 + (fb)y_0 + (fc)z_0 \ldots + (ff)t_0 &= (fn), \end{aligned}$$

where, for example,

$$\begin{aligned} (aa) \text{ denotes } & a_1^2 + a_2^2 \ldots + a_m^2, \\ (ab) \quad \text{,,} \quad & a_1b_1 + a_2b_2 \ldots + a_mb_m, \end{aligned}$$

so that $(ab)=(ba)$, &c.

* The notation, as in the previous paper, is derived from the case of six unknowns, x, y, z, w, u, t, the coefficients being denoted by a, b, c, d, e, f, with suffixes attached. All the formulæ in the paper, however, have reference to the general case of μ unknowns, $x, y, z, \ldots t$, with corresponding coefficients $a, b, c, \ldots f$.

The signs of the quantities $n_1, n_2, \ldots n_m$ have been changed, so that the equations of condition are written

$$a_1x + b_1y + c_1z \ldots + f_1t = n_1, \quad \&c.$$

instead of

$$a_1x + b_1y + c_1z \ldots + f_1t + n_1 = 0, \quad \&c.,$$

as it is somewhat more convenient to have the quantities determined by observation on the right hand side of the equations. This change is also made in Oppolzer's *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*, vol. ii. p. 313 (1880). In my previous paper Gauss's and Encke's form (*i.e.* the second of the above forms) was retained.

The values of x_0, y_0, ... are therefore

$$x_0 = \frac{\begin{vmatrix} (an), (ab), (ac), \ldots (af) \\ (bn), (bb), (bc), \ldots (bf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fn), (fb), (fc), \ldots (ff) \end{vmatrix}}{\begin{vmatrix} (aa), (ab), (ac), \ldots (af) \\ (ba), (bb), (bc), \ldots (bf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fa), (fb), (fc), \ldots (ff) \end{vmatrix}},$$

$$y_0 = \frac{\begin{vmatrix} (aa), (an), (ac), \ldots (af) \\ (ba), (bn), (bc), \ldots (bf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fa), (fn), (fc), \ldots (ff) \end{vmatrix}}{\begin{vmatrix} (aa), (ab), (ac), \ldots (af) \\ (ba), (bb), (bc), \ldots (bf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fa), (fb), (fc), \ldots (ff) \end{vmatrix}},$$

z_0 = &c.

The weights p_x, p_y, ... of x_0, y_0, ... are

$$p_x = \frac{\begin{vmatrix} (aa), (ab), \ldots (af) \\ (ba), (bb), \ldots (bf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fa), (fb), \ldots (ff) \end{vmatrix}}{\begin{vmatrix} (bb), (bc), \ldots (bf) \\ (cb), (cc), \ldots (cf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fb), (fc), \ldots (ff) \end{vmatrix}},$$

$$p_y = \frac{\begin{vmatrix} (aa), (ab), \ldots (af) \\ (ba), (bb), \ldots (bf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fa), (fb), \ldots (ff) \end{vmatrix}}{\begin{vmatrix} (aa), (ac), \ldots (af) \\ (ca), (cc), \ldots (cf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fa), (fc), \ldots (ff) \end{vmatrix}},$$

p_z = &c.;

and the sum of the squares of the residuals, viz. (vv),

$$= \frac{\begin{vmatrix} (aa), & (ab), & \dots & (af), & (an) \\ (ba), & (bb), & \dots & (bf), & (bn) \\ \dots & \dots & \dots & \dots & \dots \\ (fa), & (fb), & \dots & (ff), & (fn) \\ (na), & (nb), & \dots & (nf), & (nn) \end{vmatrix}}{\begin{vmatrix} (aa), & (ab), & \dots & (af) \\ (ba), & (bb), & \dots & (bf) \\ \dots & \dots & \dots & \dots \\ (fa), & (fb), & \dots & (ff) \end{vmatrix}}.$$

If therefore ϵ_x, ϵ_y, ... denote the mean errors of x_0, y_0, ...

$$\epsilon_x^2 = \frac{1}{m-\mu} \frac{\begin{vmatrix} (bb), & (bc), & \dots & (bf) \\ (cb), & (cc), & \dots & (cf) \\ \dots & \dots & \dots & \dots \\ (fb), & (fc), & \dots & (ff) \end{vmatrix} \begin{vmatrix} (aa), & (ab), & \dots & (af), & (an) \\ (ba), & (bb), & \dots & (bf), & (bn) \\ \dots & \dots & \dots & \dots & \dots \\ (fa), & (fb), & \dots & (ff), & (fn) \\ (na), & (nb), & \dots & (nf), & (nn) \end{vmatrix}}{\begin{vmatrix} (aa), & (ab), & \dots & (af) \\ (ba), & (bb), & \dots & (bf) \\ \dots & \dots & \dots & \dots \\ (fa), & (fb) & \dots & (ff) \end{vmatrix}^2},$$

$$\epsilon_y^2 = \frac{1}{m-\mu} \frac{\begin{vmatrix} (aa), & (ac), & \dots & (af) \\ (ca), & (cc), & \dots & (cf) \\ \dots & \dots & \dots & \dots \\ (fa), & (fc), & \dots & (ff) \end{vmatrix} \begin{vmatrix} (aa), & (ab), & \dots & (af), & (an) \\ (ba), & (bb), & \dots & (bf), & (bn) \\ \dots & \dots & \dots & \dots & \dots \\ (fa), & (fb), & \dots & (ff), & (fn) \\ (na), & (nb), & \dots & (nf), & (nn) \end{vmatrix}}{\begin{vmatrix} (aa), & (ab), & \dots & (af) \\ (ba), & (bb), & \dots & (bf) \\ \dots & \dots & \dots & \dots \\ (fa), & (fb), & \dots & (ff) \end{vmatrix}^2},$$

ϵ_z^2 = &c.;

where μ is the number of unknowns.

The form of the expressions for ϵ_x^2, ϵ_y^2, ... is remarkable; the numerators consist of the product of two determinants, of which one contains $\mu-1$, and the other $\mu+1$ rows, and the denominator is the square of a determinant of μ rows.

Denoting the last-mentioned determinant, viz.

$$\begin{vmatrix} (aa), (ab), \ldots (af) \\ (ba), (bb), \ldots (bf) \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ (fa), (fb), \ldots (ff) \end{vmatrix}$$

by ∇, then, in the numerator of the expression for ε_x^2, the first determinant is formed from ∇ by omitting the first line and column, and the second determinant is formed from ∇ by bordering it with the elements $(an), (bn), \ldots (nn), (nf), \ldots (na)$; similarly, the first determinant in the numerator of ε_y^2 is derived from ∇ by omitting the second line and second column, and so on. The second determinant and the denominator, ∇^2, are the same in all the expressions.

The foregoing values of x_0, $y_0, \ldots$ as determinants and of $p_x, p_y, \ldots$ as quotients of determinants may of course be written down at once from the equations which determine them, and no formal proof is needed.* The value of (vv) is taken from § 7 of the previous paper, and a direct proof of this result is given in the next section (§ 15). In § 6 expressions were found for the auxiliaries $(bb.1)$, $(bc.1)$, $(cc.2)$, &c., as quotients of determinants.

§ 15. The method by which it was shown in § 7 that (vv) was equal to

$$\frac{1}{\nabla} \begin{vmatrix} (aa), (ab), \ldots (af), (an) \\ (ba), (bb), \ldots (bf), (bn) \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ (fa), (fb), \ldots (ff), (fn) \\ (na), (nb), \ldots (nf), (nn) \end{vmatrix},$$

depended upon the known equality of (vv) and $(nn.6)$. The expression for (vv) may, however, be very simply established, without assuming this equality, as follows:—

$$\begin{aligned} (vv) &= \Sigma(a_1x_0 + b_1y_0 + c_1z_0 \ldots - n_1)^2, \\ &= (aa)x_0^2 + (bb)y_0^2 + \ldots + 2(ab)x_0y_0 + 2(ac)x_0z_0 + \ldots \\ &\qquad - 2(an)x_0 - 2(bn)y_0 - \ldots + (nn), \\ &= x_0\{(aa)x_0 + (ab)y_0 \ldots + (af)t_0 - (an)\} \\ &\quad + y_0\{(ba)x_0 + (bb)y_0 \ldots + (bf)t_0 - (bn)\} \\ &\quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\quad + t_0\{(fa)x_0 + (fb)y_0 \ldots + (ff)t_0 - (fn)\} \\ &\quad - (an)x_0 - (bn)y_0 \ldots - (fn)t_0 + (nn). \end{aligned}$$

* In a paper entitled "Over het gebruik van determinanten bij de methode der kleinste kwadraten," printed in the *Nieuw Archief voor Wiskunde*, Deel I. pp. 179–188 (1875), Mr. Van Geer gives the determinant values of x_0, $y_0, \ldots$ and of their weights. The determinants which form the denominators of the latter are, through some inadvertence, erroneous, although the equations from which they are derived are correctly stated.

In virtue of the normal equations which x_0, y_0, . . . satisfy, each line in this expression vanishes except the last, and therefore

$$(vv) = -(an)x_0 - (bn)y_0 \ldots - (fn)t_0 + (nn),$$

$$= -\frac{(an)}{\nabla}\begin{vmatrix} (an), & (ab), & \ldots & (af) \\ (bn), & (bb), & \ldots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fn), & (fb), & \ldots & (ff) \end{vmatrix} - \frac{(bn)}{\nabla}\begin{vmatrix} (aa), & (an), & \ldots & (af) \\ (ba), & (bn), & \ldots & (bf) \\ \cdot & \cdot & \cdot & \cdot \\ (fa), & (fn), & \ldots & (ff) \end{vmatrix} - \ldots + (nn),$$

$$= (-)^{\mu}\frac{1}{\nabla}\begin{vmatrix} (an), & (bn), & \ldots & (fn), & (nn) \\ (aa), & (ab), & \ldots & (af), & (an) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (ba), & (bb), & \ldots & (bf), & (bn) \\ (fa), & (fb), & \ldots & (ff), & (fn) \end{vmatrix},$$

$$= \frac{1}{\nabla}\begin{vmatrix} (aa), & (ab), & \ldots & (af), & (an) \\ (ba), & (bb), & \ldots & (bf), & (bn) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ (fa), & (fb), & \ldots & (ff), & (fn) \\ (na), & (nb), & \ldots & (nf), & (nn) \end{vmatrix}.$$

It will be noticed that the analysis employed in this section merely amounts to the use of the following almost obvious theorem:—taking three letters only for simplicity, if x, y, z be given by the equations

$$\alpha_1 x + \beta_1 y + \gamma_1 z = h_1,$$
$$\alpha_2 x + \beta_2 y + \gamma_2 z = h_2,$$
$$\alpha_3 x + \beta_3 y + \gamma_3 z = h_3,$$

then

$$px + qy + rz - k = \frac{\begin{vmatrix} p, & q, & r, & k \\ \alpha_1, & \beta_1, & \gamma_1, & h_1 \\ \alpha_2, & \beta_2, & \gamma_2, & h_2 \\ \alpha_3, & \beta_3, & \gamma_3, & h_3 \end{vmatrix}}{\begin{vmatrix} \alpha_1, & \beta_1, & \gamma_1 \\ \alpha_2, & \beta_2, & \gamma_2 \\ \alpha_3, & \beta_3, & \gamma_3 \end{vmatrix}}.$$

If the signs of n_1, n_2, . . . n_m be changed, the signs of the expressions x_0, y_0, . . . are changed, but the values of (vv), p_x, p_y, . . . ϵ_x^2, ϵ_y^2, . . . remain unaltered.

§16. *Theorem*: The determinant ∇, viz.

$$\left|\begin{array}{l} (aa),\ (ab),\ \ldots\ (af) \\ (ba),\ (bb),\ \ldots\ (bf) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (fa),\ (fb),\ \ldots\ (ff) \end{array}\right| \quad \ldots\ldots (1)$$

is equal to the sum of the squares of p determinants, p denoting the number of combinations of m things μ together.

To prove this, write at length the determinant ∇,

$$\left|\begin{array}{llll} a_1^2 + a_2^2 \ \ldots + a_m^2\ , & b_1a_1 \ldots + b_ma_m\ , & \ldots, & f_1a_1 \ldots + f_ma_m \\ a_1b_1 + a_2b_2 \ldots + a_mb_m, & b_1^2 \ \ldots + b_m^2\ , & \ldots, & f_1b_1 \ldots + f_mb_m \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & & & \\ a_1f_1 + a_2f_2 \ldots + a_mf_m, & b_1f_1 \ldots + b_mf_m\ , & \ldots, & f_1^2 \ \ldots + f_m^2 \end{array}\right|$$

and from the $[m]^\mu$ determinants of which this is the sum select that which is formed from the first terms of the constituents of the first column, the second terms of the second column, . . . and the μ-th terms of the last column : this determinant

$$= a_1b_2 \ldots f_\mu \left|\begin{array}{l} a_1,\ a_2,\ \ldots\ a_\mu \\ b_1,\ b_2,\ \ldots\ b_\mu \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ f_1,\ f_2,\ \ldots\ f_\mu \end{array}\right|.$$

Similarly the determinant formed from the second terms of the first column, the first terms of the second column, and the same constituents as before in the case of the other columns, is

$$a_2b_1c_3 \ldots f_\mu \left|\begin{array}{l} a_2,\ a_1,\ a_3,\ \ldots\ a_\mu \\ b_2,\ b_1,\ b_3,\ \ldots\ b_\mu \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ f_2,\ f_1,\ f_3,\ \ldots\ f_\mu \end{array}\right|,$$

which is

$$= -a_2b_1c_3 \ldots f_\mu \left|\begin{array}{l} a_1,\ a_2,\ \ldots\ a_\mu \\ b_1,\ b_2,\ \ldots\ b_\mu \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ f_1,\ f_2,\ \ldots\ f_\mu \end{array}\right|,$$

and in this way it is easily seen that the coefficient of the determinant

$$\left|\begin{array}{l} a_1,\ a_2,\ \ldots\ a_\mu \\ b_1,\ b_2,\ \ldots\ b_\mu \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ f_1,\ f_2,\ \ldots\ f_\mu \end{array}\right| \quad \ldots\ldots (2)$$

is $\Sigma \pm a_1 b_2 c_3 \ldots f_\mu$, the summation referring to all permutations of the suffixes 1, 2, . . . μ: but this quantity is itself equal to the determinant (2), so that the resulting expression is equal to the square of (2), and, interchanging the lines and columns in the determinants of the form (2), we have the result

$$\begin{vmatrix} (aa), (ab), \ldots (af) \\ (ba), (bb), \ldots (bf) \\ \ldots\ldots\ldots \\ (fa), (fb), \ldots (ff) \end{vmatrix} = \Sigma \begin{vmatrix} a_1, b_1, \ldots f_1 \\ a_2, b_2, \ldots f_2 \\ \ldots\ldots \\ a_\mu, b_\mu, \ldots f_\mu \end{vmatrix}^2 \quad \ldots\ldots (3)$$

where the summation symbol refers to the suffixes which in the different determinants are the $[m]^\mu \div [m]^\mu$ sets of μ numbers that can be formed from the m numbers 1, 2, . . . m.

Using a recognised notation, the determinant (2) may be conveniently written

$$(a_1 b_2 \ldots f_\mu),$$

and the theorem is that the determinant (1) is equal to $\Sigma\,(a_1 b_2 \ldots f_\mu)^2$.

§17. It can be easily seen, in the same manner, that the determinant

$$\begin{vmatrix} (an), (ab), (ac), \ldots (af) \\ (bn), (bb), (bc), \ldots (bf) \\ \ldots\ldots\ldots \\ (fn), (fb), (fc), \ldots (ff) \end{vmatrix} \quad \ldots\ldots (4)$$

is equal to

$$\Sigma \begin{vmatrix} a_1, b_1, \ldots f_1 \\ a_2, b_2, \ldots f_2 \\ \ldots\ldots \\ a_\mu, b_\mu, \ldots f_\mu \end{vmatrix} \begin{vmatrix} n_1, b_1, \ldots f_1 \\ n_2, b_2, \ldots f_2 \\ \ldots\ldots \\ n_\mu, b_\mu, \ldots f_\mu \end{vmatrix}$$

or, as this expression may be written,

$$\Sigma (a_1 b_2 \ldots f_\mu)(n_1 b_2 \ldots f_\mu).$$

the Σ having the same meaning as at the end of the last section, and the number of terms—*i.e.* of products of pairs of determinants—being as before $= [m]^\mu \div [\mu]^\mu$.

§18. The values of x_0, y_0, . . . are therefore

$$x_0 = \frac{\Sigma (a_1 b_2 \ldots f_\mu)(n_1 b_2 \ldots f_\mu)}{\Sigma (a_1 b_2 \ldots f_\mu)^2},$$

$$y_0 = \frac{\Sigma (a_1 b_2 \ldots f_\mu)(a_1 n_2 \ldots f_\mu)}{\Sigma (a_1 b_2 \ldots f_\mu)^2},$$

$$z_0 = \&c.;$$

and it thus appears that the values of $x, y, \ldots$ given by the method of least squares are in fact those obtained by combining linearly in the manner indicated the values found by solving each set of μ equations which can be formed from the m given equations. Expressing this more in detail, we have the following rule:—

From the m equations of condition we can form p sets of μ equations, p denoting for brevity $[m]^{\mu} \div [\mu]^{\mu}$. Solve each of these sets of equations by the determinant method, and let the resulting values be

$$\text{for } x, \quad \frac{\alpha_1}{\lambda_1}, \frac{\alpha_2}{\lambda_2}, \ldots \frac{\alpha_p}{\lambda_p},$$

$$\text{for } y, \quad \frac{\beta_1}{\lambda_1}, \frac{\beta_2}{\lambda_2}, \ldots \frac{\beta_p}{\lambda_p},$$

&c.,

where $\alpha_1, \ldots \beta_1, \ldots \lambda_1, \ldots$ are the actual determinants which occur in the solutions (*i.e.* so that any factors common to both numerator and denominator in any of the fractions are not to be thrown out); then

$$x_0 = \frac{\lambda_1\alpha_1 + \lambda_2\alpha_2 \ldots + \lambda_p\alpha_p}{\lambda_1^2 + \lambda_2^2 \ldots + \lambda_p^2},$$

$$y_0 = \frac{\lambda_1\beta_1 + \lambda_2\beta_2 \ldots + \lambda_p\beta_p}{\lambda_1^2 + \lambda_2^2 \ldots + \lambda_p^2},$$

$$z_0 = \&c.$$

The rule may also be stated in a slightly different manner thus:— solve each set of equations and let the system of values be

$$\text{for } x, \quad A_1, A_2, \ldots A_p,$$

$$\text{for } y, \quad B_1, B_2, \ldots B_p,$$

&c.;

then

$$x_0 = \frac{\lambda_1^2 A_1 + \lambda_2^2 A_2 \ldots + \lambda_p^2 A_p}{\lambda_1^2 + \lambda_2^2 \ldots + \lambda_p^2},$$

$$y_0 = \frac{\lambda_1^2 B_1 + \lambda_2^2 B_2 \ldots + \lambda_p^2 B_p}{\lambda_1^2 + \lambda_2^2 \ldots + \lambda_p^2},$$

$$z_0 = \&c.,$$

where $\lambda_1, \lambda_2, \ldots \lambda_p$ are as before: that is, they are the determinants whose constituents are the coefficients which occur on the left-hand side of the different sets of equations.

§19. To illustrate the formulæ, take the simple case of $m=4, \mu=3$.

By §16,

$$\begin{vmatrix} (aa), & (ab), & (ac) \\ (ba), & (bb), & (bc) \\ (ca), & (cb), & (cc) \end{vmatrix} = (a_1b_2c_3)^2 + (a_1b_2c_4)^2 + (a_1b_3c_4)^2 + (a_2b_3c_4)^2,$$

by §17,

$$\begin{vmatrix} (an), & (ab), & (ac) \\ (bn), & (bb), & (bc) \\ (cn), & (cb), & (cc) \end{vmatrix} = (a_1b_2c_3)(n_1b_2c_3) + (a_1b_2c_4)(n_1b_2c_4) + (a_1b_3c_4)(n_1b_3c_4) + (a_2b_3c_4)(n_2b_3c_4),$$

and by §18,

$$x_0 = \frac{(a_1b_2c_3)(n_1b_2c_3) + (a_1b_2c_4)(n_1b_2c_4) + (a_1b_3c_4)(n_1b_3c_4) + (a_2b_3c_4)(n_2b_3c_4)}{(a_1b_2c_3)^2 + (a_1b_2c_4)^2 + (a_1b_3c_4)^2 + (a_2b_3c_4)^2},$$

$$y_0 = \frac{(a_1b_2c_3)(a_1n_2c_3) + (a_1b_2c_4)(a_1n_2c_4) + (a_1b_3c_4)(a_1n_3c_4) + (a_2b_3c_4)(a_2n_3c_4)}{(a_1b_2c_3)^2 + (a_1b_2c_4)^2 + (a_1b_3c_4)^2 + (a_2b_3c_4)^2},$$

$$z_0 = \frac{(a_1b_2c_3)(a_1b_2n_3) + (a_1b_2c_4)(a_1b_2n_4) + (a_1b_3c_4)(a_1b_3n_4) + (a_2b_3c_4)(a_2b_3n_4)}{(a_1b_2c_3)^2 + (a_1b_2c_4)^2 + (a_1b_3c_4)^2 + (a_2b_3c_4)^2}.$$

§20. As a numerical example of the process, I now consider the system of four equations which Gauss himself employed to illustrate the method of least squares, and which has generally been adopted as the standard example by writers on the subject. These equations are

$$\left.\begin{aligned} x - y + 2z &= 3 \quad \text{(i)} \\ 3x + 2y - 5z &= 5 \quad \text{(ii)} \\ 4x + y + 4z &= 21 \quad \text{(iii)} \\ -x + 3y + 3z &= 14 \quad \text{(iv)} \end{aligned}\right\};$$

and the normal equations derived from them are

$$\left.\begin{aligned} 27x_0 + 6y_0 &= 88 \\ 6x_0 + 15y_0 + z_0 &= 70 \\ y_0 + 54z_0 &= 107 \end{aligned}\right\},$$

which give

$$x_0 = \frac{49154}{19899}, \quad y_0 = \frac{2617}{737}, \quad z_0 = \frac{12707}{6633}.$$

Taking the first three equations (i), (ii), (iii), the values of x, y, z, given by them are

$$x = \frac{P_1}{P}, \quad y = \frac{P_2}{P}, \quad z = \frac{P_3}{P},$$

where

$$P = \begin{vmatrix} 1, & -1, & 2 \\ 3, & 2, & -5 \\ 4, & 1, & 4 \end{vmatrix} = 35,$$

$$P_1 = \begin{vmatrix} 3, & -1, & 2 \\ 5, & 2, & -5 \\ 21, & 1, & 4 \end{vmatrix} = 90,$$

$$P_2 = \begin{vmatrix} 1, & 3, & 2 \\ 3, & 5, & -5 \\ 4, & 21, & 4 \end{vmatrix} = 115,$$

$$P_3 = \begin{vmatrix} 1, & -1, & 3 \\ 3, & 2, & 5 \\ 4, & 1, & 21 \end{vmatrix} = 65.$$

Similarly solving equations (i), (ii), (iv),

$$x = \frac{Q_1}{Q}, \quad y = \frac{Q_2}{Q}, \quad z = \frac{Q_3}{Q},$$

where the values of the determinants are

$$Q = \begin{vmatrix} 1, & -1, & 2 \\ 3, & 2, & -5 \\ -1, & 3, & 3 \end{vmatrix} = 47,$$

$$Q_1 = 122, \quad Q_2 = 167, \quad Q_3 = 93.$$

Solving the systems (i), (iii), (iv), and (ii), (iii), (iv), and denoting the values of x, y, z, by

$$\frac{R_1}{R}, \quad \frac{R_2}{R}, \quad \frac{R_3}{R}$$

and

$$\frac{S_1}{S}, \quad \frac{S_2}{S}, \quad \frac{S_3}{S},$$

the values of the determinants $R, R_1, \ldots S, S_1, \ldots$ are found to be

$$R = 33, \quad R_1 = 78, \quad R_2 = 113, \quad R_3 = 67$$
$$S = -124, \quad S_1 = -304, \quad S_2 = -444, \quad S_3 = -236.$$

The unreduced values obtained from the four sets of equations are thus

$$\text{for } x, \quad \frac{90}{35}, \ \frac{122}{47}, \ \frac{78}{33}, \ \frac{304}{124},$$

$$\text{for } y, \quad \frac{115}{35}, \ \frac{167}{47}, \ \frac{113}{33}, \ \frac{444}{124},$$

$$\text{for } z, \quad \frac{65}{35}, \ \frac{93}{47}, \ \frac{67}{33}, \ \frac{236}{124},$$

and therefore

$$x_0 = \frac{90\times35+122\times47+78\times33+304\times124}{(35)^2+(47)^2+(33)^2+(124)^2},$$

$$y_0 = \frac{115\times35+167\times47+113\times33+444\times124}{(35)^2+(47)^2+(33)^2+(124)^2},$$

$$z_0 = \frac{65\times35+93\times47+67\times33+236\times124}{(35)^2+(47)^2+(33)^2+(124)^2},$$

giving

$$x_0 = \frac{49154}{19899}, \quad y_0 = \frac{70659}{19899}, \quad z_0 = \frac{38121}{19899},$$

which agree with the values given by the normal equations.

In general, if the values obtained by solving the equations be given in any form such as, *e.g.*, vulgar fractions in their lowest terms, so that the values of x are

$$\frac{18}{7}, \ \frac{122}{47}, \ \frac{26}{11}, \ \frac{76}{31},$$

then these quantities are to have the respective weights

$$P^2, \quad Q^2, \quad R^2, \quad S^2,$$

viz.

$$x_0 = \frac{\frac{18}{7}\times(35)^2+\frac{122}{47}\times(47)^2+\frac{26}{11}\times(33)^2+\frac{76}{31}\times(124)^2}{(35)^2+(47)^2+(33)^2+(124)^2},$$

and the weights are the same in the case of the corresponding values of y and z.

§21. In the absence of any method such as that of least squares, if we had to determine the best values of x, y, . . . from a system of m linear equations, it would be natural to first solve every set of μ equations which could be formed from the m equations, and to compare the different values of x, y, . . . thus obtained. The question to be decided would be how to combine the different values, and perhaps the method which would first suggest itself would be to take the arithmetic mean of the values found for x as the adopted value of x, and similarly in the case of

$y, z, \ldots$; it is clear, however, that this mode of treatment would not be satisfactory, as certain of the sets of equations would be better suited for the accurate determination of $x, y, \ldots$ than others.

It appears from § 18 and 22 that in proceeding according to the method of least squares we assign to each system of values of $x, y, \ldots$ a weight proportional to the square of the determinant whose constituents are the coefficients in the set of μ equations from which the values are derived.

It is evident that this determinant affords a good measure of the precision with which $x, y, \ldots$ are determined by the set of equations, representing as it does the common denominator in the values of these quantities. In the method of least squares the square of the determinant is taken as the weight, and in consequence the sign of the determinant is immaterial, and the common denominator of $x_0, y_0, \ldots$, being a sum of squares, is always positive.

§ 22. The determinant which forms the numerator of (vv), viz.

$$\left|\begin{array}{l} (aa),\ (ab),\ \ldots\ (af),\ (an) \\ (ba),\ (bb),\ \ldots\ (bf),\ (bn) \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ (fa),\ (fb),\ \ldots\ (ff),\ (fn) \\ (na),\ (nb),\ \ldots\ (nf),\ (nn) \end{array}\right| \quad (\mu+1 \text{ rows}) \ldots\ldots (5)$$

is of the same form as ∇ and only differs from it by including the letters n: the determinant (5) is therefore

$$= \Sigma \left|\begin{array}{llll} a_1, & b_1, & \ldots f_1, & n_1, \\ a_2, & b_2, & \ldots f_2, & n_2, \\ \ldots & \ldots & \ldots & \ldots \\ a_\mu, & b_\mu, & \ldots f_\mu, & n_\mu \\ a_{\mu+1}, & b_{\mu+1}, & \ldots f_{\mu+1}, & n_{\mu+1} \end{array}\right|^2 \ldots\ldots (6)$$

i.e. it is equal to the sum of the squares of all the determinants whose constituents are the coefficients and right-hand members in the different sets of $(\mu+1)$ equations which can be formed from the m equations; the number of such determinants is $[m]^{\mu+1} \div [\mu+1]^{\mu+1}$. It thus appears that (vv) cannot vanish unless every one of these determinants is equal to zero—*i.e.* unless the m equations are all consistent with one another, and equivalent to only μ independent equations. If $(vv)=0$, the mean (or probable) errors of $x_0, y_0, \ldots$ also vanish; and this is as it should be, for it is clear that the mean (or probable) errors of $x_0, y_0, \ldots$ can only be zero when the m equations determine $x, y, \ldots$ uniquely—*i.e.* are equivalent to only μ independent equations.

§ 23. If the number of equations exceeds the number of un-

knowns by unity—*i.e.* if $m=\mu+1$—then the expression (6) consists of only a single determinant, so that (vv) is a complete square, and therefore $\sqrt{(vv)}$ is a linear function of $n_1, n_2, \ldots n_{\mu+1}$, its value being

$$\frac{1}{\nabla}\begin{vmatrix} a_1, & b_1, & \ldots f_1, & n_1 \\ a_2, & b_2, & \ldots f_2, & n_2 \\ \ldots & \ldots & \ldots & \ldots \\ a_\mu, & b_\mu, & \ldots f_\mu, & n_\mu \\ a_{\mu+1}, & b_{\mu+1}, & \ldots f_{\mu+1}, & n_{\mu+1} \end{vmatrix}.$$

If in this determinant capital letters be used to denote the minors of the corresponding italic letters, then

$$\sqrt{(vv)} = \frac{n_1N_1+n_2N_2\ldots+n_{\mu+1}N_{\mu+1}}{\sqrt{(N_1^2+N_2^2\ldots N^2{}_{\mu+1})}},$$

$$x_0 = -\frac{A_1N_1+A_2N_2\ldots+A_{\mu+1}N_{\mu+1}}{N_1^2+N_2^2\ldots+N^2{}_{\mu+1}},$$

$$y_0 = -\frac{B_1N_1+B_2N_2\ldots+B_{\mu+1}N_{\mu+}}{N_1^2+N_2^2\ldots+N^2_{\mu+1}},$$

$$z_0 = \&c.$$

In the case of the foregoing example

$$\sqrt{(vv)} = \frac{1}{\sqrt{\nabla}}\begin{vmatrix} 1, & -1, & 2, & 3 \\ 3, & 2, & -5, & 5 \\ 4, & 1, & 4, & 21 \\ -1, & 3, & 3, & 14 \end{vmatrix}$$

$$= \frac{14P-21Q+5R-3S}{\sqrt{(P^2+Q^2+R^2+S^2)}}$$

$$= \frac{14\times35-21\times47+5\times33+3\times124}{\sqrt{\{(35)^2+(47)^2+(33)^2+(124)^2\}}}$$

$$= \frac{40}{\sqrt{(19899)}}.$$

§ 24. The determinants which form the denominators of $p_x, p_y, \ldots$ are also sums of squares of determinants; for example, the denominator of p_x, viz:

$$\begin{vmatrix} (bb), & (bc), & \ldots & (bf) \\ (cb), & (cc), & \ldots & (cf) \\ \cdot & \cdot & \cdot & \cdot \\ (fb), & (fe), & \ldots & (ff) \end{vmatrix} \quad (\mu-1 \text{ rows}) \quad \ldots\ldots (7)$$

$$= \Sigma \begin{vmatrix} b_1, & c_1, & \ldots & f_1 \\ b_2, & c_2, & \ldots & f_2 \\ \cdot & \cdot & \cdot & \cdot \\ b_{\mu-1}, & c_{\mu-1}, & \ldots & f_{\mu-1} \end{vmatrix}^2 \ldots\ldots (8)$$

the number of determinants being $[m]^{\mu-1} \div [\mu-1]^{\mu-1}$.

Thus, in the example, the denominator of p_x

$$= \begin{vmatrix} -1, & 2 \\ 2, & -5 \end{vmatrix}^2 + \begin{vmatrix} -1, & 2 \\ 1, & 4 \end{vmatrix}^2 + \begin{vmatrix} -1, & 2 \\ 3, & 3 \end{vmatrix}^2$$

$$+ \begin{vmatrix} 2, & -5 \\ 1, & 4 \end{vmatrix}^2 + \begin{vmatrix} 2, & -5 \\ 3, & 3 \end{vmatrix}^2 + \begin{vmatrix} 1, & 4 \\ 3, & 3 \end{vmatrix}^2$$

$$= 1 + 36 + 81 + 169 + 441 + 81$$

$$= 809,$$

and therefore

$$p_x = \frac{19899}{809}.$$

It may be observed that if the determinant which forms the denominator in any of the expressions for the weights is equal to zero, so also is the numerator, for if each of the determinants in (8) vanishes, then each of the determinants which form the right-hand side of (3) vanishes also.

§ 25. Since all the determinants involved in the statement of results in § 14 are equal to sums of squares, it follows that they can never be negative. In § 5 it was shown that all the products $(aa)(bb.1)$, $(aa)(bb.1)(cc.2)$, &c., are of the form (1), and it is therefore evident that the auxiliaries $(bb.1)$, $(cc.2)$, &c., cannot be negative. A shorter proof of this fact was, however, given in § 7.

§ 26. The case, $\mu=2$, of the theorem in § 16 is a very well known result; but the only place I know of in which the general theorem is enunciated occurs in an investigation by Professor C. Niven of the vibrations of a dynamical system where the particles are subject to small frictional forces, printed in the "Cambridge Senate-House Problems and Riders for 1878," pp. 188–191. The proof given in § 16 is the same as Professor Niven's.

Professor Cayley, to whom I communicated the results of §§ 16–18 some time ago was, I found, already acquainted with them; but it seems possible that they may not have been published before, as it would not be easy to express them without the aid of determinants, and the only paper I have met with in which an explicit use of determinants is made in connection with the solution of equations in the method of least squares is that of Mr. Van Geer referred to in the note in § 14.

On the Possible Performance of an Object-Glass for Star-Gazing.
By Edward Sang, Esq.

It having been proposed to compute the curvatures for an object-glass, with a view to obtaining the least possible aberration in the image of a star, the preliminary question arose as to how the computations should be conducted.

In the preparation of formulæ for the amount of spherical aberration, the sines and cosines of arcs are represented by two or three terms of the series which truly express them, and therefore such formulæ are only approximative, and the results obtained by them are to be regarded as guides to more accurate determinations. Our ultimate resort is to trace strictly the course of each pencil of light. When the thicknesses are taken into account, the application of the formulæ becomes as laborious as the direct trigonometrical calculation itself; wherefore it was determined to follow the trigonometrical method throughout.

The computation thus takes the form of a series of trials applicable only to the particular case in hand, and we have so to arrange these trials as to make them exhaustive, and so also as to throw light on analogous cases.

The proposition as it occurs in practice is this:—"Given two discs of glass, to construct of them an object-glass which shall give the best possible result." In the present instance that result is to be the formation of the image of an exceedingly minute luminous object; the correlative matters of the flatness of the field of view and of the performance towards the edge of that field not being taken into account. Now, the refractions by the two kinds of glass are data in the problem and fix the amount of the secondary chromatic aberration; wherefore our enquiry must be mainly directed to that part of the total error, which depends on the sphericity of the surfaces—that being the only matter under the control of the constructor.

The case actually proposed was to make an aplanatic combination from two discs, one of *hard-crown*, the other of *dense flint* glass, having an aperture of 7·5 inches, with a thickness in the rough of ·75; the indices of refraction being given in Chance's list as under, and the focal distance to be about 100 inches.

Glass (Chance's).	Density.	C	D	F	G
Hard Crown	2·485	1·5146	1·5172	1·5232	1·5280
Soft Crown	2·55	1·5119	1·5146	1·5210	1·5263
Light Flint	3·21	1·5700	1·5740	1·5839	1·5922
Dense Flint	3·66	1·6175	1·6224	1·6348	1·6453
Extra Dense	3·85	1·6450	1·6504	1·6643	1·6761

The investigation was divided into two parts: one in which the extreme bands C and G alone were considered; the other, in which all the four were taken into account.

The first step was to compute, as for an absolutely thin lens, the curvatures needed to bring together the images for the extreme bands C and G, the result being:—

$$\frac{1}{r_1} - \frac{1}{r_2} = -\cdot0460\ 9225,$$

$$\frac{1}{r_3} - \frac{1}{r_4} = +\cdot0222\ 1714;$$

so that the thickness of the crown-glass lens may conveniently be fixed at ·70, that of the flint-glass one at ·54, measured on the axis of the system.

These equations show no connection between the second and third curvatures; the internal surfaces may be made coincident, or the glasses may have a lens of air between them, which may be either convex or concave; hence our trials must be arranged in sets, according to the arbitrarily assumed character of this intermediate lens.

Having established the relation between the second and first curvatures, indicated by the equation

$$\frac{1}{r_2} = \frac{1}{r_1} + \cdot0460\ 9225,$$

and that between the third and second curvatures by

$$\frac{1}{r_3} = \frac{1}{r_2} + \text{arbitrary constant},$$

the fourth curvature has to be computed so as to make the foci for the bands C and G coincident for those pencils of light which pass close to the axis; this computation is so simple as to need no remark. The influence of the thicknesses of the lenses is seen in the resulting curvature and position of the focus. In this way the assumption of the first curvature determines the whole arrangement in any one of the sets of trials.

It remains for us to compute the aberrations for those pencils of light which pass at the margin of the lens.

This computation involves only the very simplest cases of the resolution of triangles, and would need no special notice but for the circumstance that, in consequence of the angles being small,

the interpolation of the arc from its logarithmic sine, or contrariwise, is laborious. With Taylor's table to each second of arc, the labour is very great. In the reprint of Taylor's given in Shortrede's Tables, I caused to be inserted tables of proportional parts, but even these fail to give us help, because for our purpose second and even third differences are needed. To meet the requirements of the case, I changed entirely the plan of operation, and, measuring the arcs by the same unit as their sines, constructed a canon of logarithmic sines for each ten-thousandth part of the radius up to ·3000. In this way the interpolations were rendered incomparably more easy, and the accuracy was correspondingly augmented so that the results have all the precision that is attainable by the use of seven-place tables.

It was natural to begin our trials with the second and third surfaces coincident. Assuming then $r_2 = r_3$ and taking for $\frac{1}{r_1}$ values ranging from $+\cdot005$ to $-\cdot045$, the distances of the foci from the fourth surface were computed for the achromatised central pencil, and for the marginal pencils corresponding to the bands C and G. The results are shown in the accompanying figure 1, enlarged to make the parts visible.

The numbers on the horizontal scale show the focal distances in inches (enlarged two times); those on the vertical scale, the curvatures, $\frac{1}{r_1}$, of the first surface, + indicating concavity toward the object, − indicating convexity.

The vertical line at 100 shows the position of the achromatised focus for an absolutely thin lens; the strong curved line with its convexity to the left shows the focus for the central achromatised ray of the actual lens, and the two lines convex to the right indicate the foci for the marginal pencils of the bands C and G. These are well apart for the case

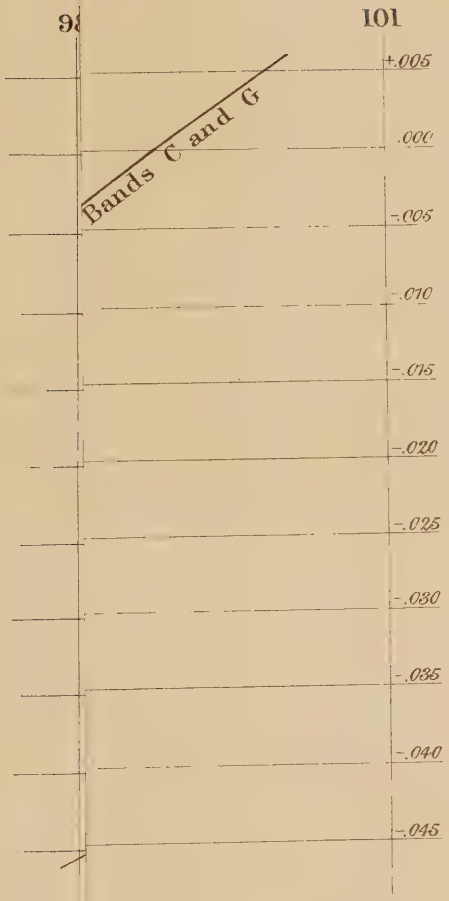
$$\frac{1}{r_1} = +\cdot005;$$

they become closer and cross each other about

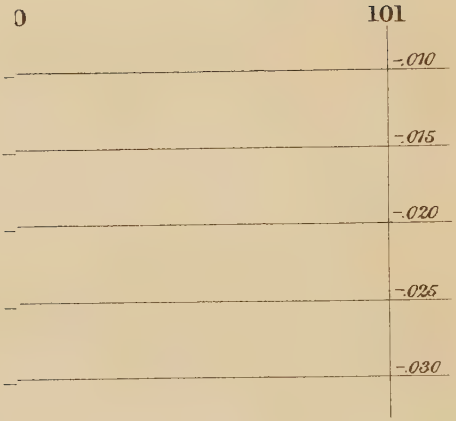
$$\frac{1}{r_1} = -\cdot040,$$

but their separation is too small to be shown even on the enlarged scale. The horizontal distances between one of these curves and the curve for the central pencil give the respective amounts of longitudinal aberration, and the points of crossing show the conditions for aplanatism for the particular band. Thus the spherical aberration of the band G is *nil* when

$$\frac{1}{r_1} = -\cdot0065,$$



Spottiswo


0
101
-.010
-.015
-.020
-.025
-.030

that for band C when

$$\frac{1}{r_1} = -\cdot0078.$$

The marginal line for C again crosses at $-\cdot0230$, that for G at $-\cdot0238$: so that our problem has two approximate solutions, neither of them satisfactory; the separation of the foci for the marginal pencils C and G being in the latter solution $\cdot0296$ or about $\frac{1}{34}$th of an inch.

Having failed to obtain complete aplanatism with the interior surfaces coincident, we proceed to try the effect of a separation. Assuming as the basis of our second set of trials

$$\frac{1}{r_2} = \frac{1}{r_2} + \cdot001,$$

and making the lenses touch at the edge, we get a separation of $\cdot007$ at the centre, and allow for this in our calculations; it is not necessary to carry these beyond the limits

$$\frac{1}{r_1} = -\cdot020 \quad \text{and} \quad \frac{1}{r_1} = -\cdot030.$$

The results are shown in the lower part of the second figure. Here the line for the central achromatised pencil is slightly, while the lines for the marginal pencils are considerably removed from the lens, so that if the curves were continued, the crossings would be much more apart than in the first figure. The separation, also, of the foci for the marginal pencils C and G is somewhat greater than before, and it is needless to proceed further in this direction.

On reversing the mode of variation by supposing

$$\frac{1}{r_3} = \frac{1}{r_2} - \cdot001,$$

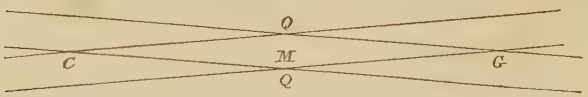
the lenses are made to touch at the centre, and their edges would need to be kept apart by means of a thin ring. The results are shown in the upper part of the second figure and are contained within the limits

$$\frac{1}{r_1} = -\cdot010 \quad \text{and} \quad \frac{1}{r_1} = -\cdot020.$$

The central line is here nearer to the lens; the marginal lines, however, much more so, so much that they do not cross the central line at all, and thus, under this supposition, we cannot correct the spherical aberration. For the purpose of comparing

these results the second figure is made in the same form as the first one. The contrast shows us that for these two kinds of glass the best result is to be got (best in so far as the bands C and G are concerned) by making the curvature of the first surface about −·0238, with the second and third surfaces coincident; in which case the fourth surface is sensibly flat.

FIG. 3.



If the curvatures be so adjusted as that the focus for the central achromatised pencil be midway between the foci for the marginal pencils C and G, as shown in figure 3, and if from C and G we draw lines to the edge of the lens, which lines have, in this case, an inclination of ·0375 to the axis, the least section of the compound cones thus formed will be on the focal plane passing through M. Now, the distance C G is ·0296; wherefore the radius M Q of this least section is

$$\cdot 0148 \times \cdot 0375 = \cdot 000555 :$$

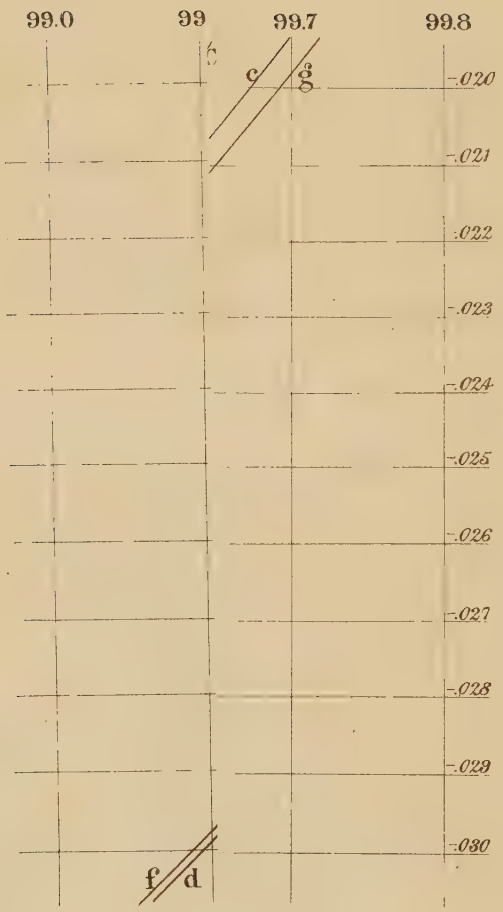
that is, its diameter will be about the 900th part of an inch, in our case. This subtends, at the distance of 100 inches, an angle of 2″·29 of the ancient division of the quadrant.

If, instead of making the central foci C and G to agree, we arrange the curvatures so as to place G before C to the extent ·0148, the central foci for C and G will agree, alternately, with the marginal foci for G and C, and the diameter of the above false disc will be halved. Thus we see that, when the aperture of the lens is ·075 of its focal length, the best performance attainable with these two kinds of glass and for the bands C and G, gives a false disc 1″·145 in diameter.

Hitherto we have considered only the extreme bands C and G; the bringing of these together does not necessarily produce achromatism, nor, indeed, can we expect that a better result might not have been reached by so treating some other pair. In combining more than two bands we cannot bring all the foci into one, and the best effect is to be got by making the sum of the squares of their mutual distances as small as possible. Also, if we desire to take into consideration the breadths and intensities of the parts of the spectrum represented by the various bands, we must attach weight-coefficients to them, so that if C represent the focus, and c the intensity of the band C, and similarly of the others, the sum

$$cd \cdot \mathrm{CD}^2 + cf \cdot \mathrm{CF}^2 + cg \cdot \mathrm{CG}^2 + df \cdot \mathrm{DF}^2 + dg \cdot \mathrm{DG}^2 + fg \quad \mathrm{FG}^2$$

is to be made a minimum.



Spottiswoode & C.° Lith. London

There is no difficulty in making this calculation for the central pencil. For our present purpose it may suffice to regard the four bands as of equal intensity; and thus we may substitute for the condition $CG^2=0$ this improved one

$$CD^2+CF^2+CG^2+DF^2+DG^2+FG^2 = \text{minimum}.$$

Proceeding exactly as before, we obtain for the conditions of optimism in a thin lens, the equations

$$\frac{1}{r_1}-\frac{1}{r_2} = -\cdot0459\ 2642,$$

$$\frac{1}{r_2}-\frac{1}{r_4} = +\cdot0220\ 8918,$$

not differing much from those formerly used.

Using the first of these equations with the same thicknesses, ·70 and ·54, as before; making the interior surfaces coincident, and computing the fourth surface so as to produce the optimum effect for the central pencils; and thereafter calculating the positions of the foci for the marginal pencils, we obtain the results exhibited in figure 4. Here the longitudinal scale has been magnified eight times: that is, each inch in the drawing represents one-eighth part of an inch in focal length. Of the eight lines shown in the figure, the four marked with capital letters indicate the foci for the central pencils of the several bands; those marked in small letters serve for the marginal pencils.

A glance at this diagram shows that the best effect is to be got with a front curvature between −·02315, where the line *gg* intersects CC, and −·02330 where *ff* cuts FF. The interval between the foci for the marginal pencils of the bands F and G is then ·1259, so that the diameter of the false disc is ·004721 which subtends at the lens an angle of 9″·8—say ten seconds.

We might now proceed to make a difference between the curvatures of the second and third surfaces, but seeing that the chromatic aberration much exceeds the error from sphericity, and seeing how closely the present resembles the former case, we can hardly expect any improvement, and thus, for these two kinds of glass, we may hold that the diameter of the false image of a minute point cannot be made less than ten seconds.

Although these enquiries have had special reference to one particular combination of glasses, they lead to some general conclusions. The residual error is seen to arise from two sources: the first and the most important being in the dispersive powers, the second in the sphericity of the surfaces of the lenses; which second is only less important than the other because it is, to a certain degree, under our control.

It is, then, of the utmost importance to select from among the different kinds of glass that are available that pair which gives the greatest concentration of the foci; and our endeavours towards the improvement of the refracting telescope should be mainly directed to the dispersive powers of the glasses. If, in our first example, the indices for the bands D and F had been such as to bring all the four foci for the central pencil together, the false disc would have only been $1''{\cdot}14$, or about one-eighth part of what, in the actual state of matters, is possible.

The importance of a proper choice among the kinds of glass may be seen by comparing the results from different combinations of the crown and flint glasses given in Chance's list. The following table shows, for an achromatic thin lens of 100 inches in focal distance, the mean longitudinal aberration from the mean focus, for the six combinations.

Mean Longitudinal Dispersion from Mean Focus.

	Hard Crown.	Soft Crown.
Light Flint	·05782	·03296
Dense Flint	·05712	·03868
Extra Dense	·05828	·04204

In the general problem "to obtain the best result from two given discs of glass," the focal length has to be considered. On computing, in several cases, the spherical aberration for pencils intermediate between the centre and the margin of the lens, these were found—as, indeed, was to have been expected—to be almost exactly proportional to the square of the distance of the incident ray from the centre. If, then, keeping the aperture unchanged, we augment the focal distance in any ratio, we reduce the longitudinal aberration in the same ratio; with twice the focal length the longitudinal spherical aberration is halved; but the inclination to the axis is also halved, wherefore the lateral aberration is reduced four times; consequently, since the distance is doubled, the angle subtended by the false disc is reduced to one-eighth part: that is, in the triplicate ratio of the focal length. So much for the spherical aberration.

On augmenting the focal length, the longitudinal chromatic aberration is augmented in the same ratio; wherefore the lateral aberration is unchanged, and the angle subtended by it is reduced in the ratio of the focal length; hence in this respect also it is advantageous to have a long telescope.

But every bit of glass is liable to internal irregularities, which the manufacturer takes many precautions to remove or to palliate. A ray of light in proceeding through the lens suffers deflection at each change of consistency, and the total effect is an indistinctness or blur in the image. This blur preserves the same angular extent whatever may be the focal length.

Hence the total imperfection of the image consists of three parts: one, depending on the quality of the glass, constant; a second, proportional inversely to the focal length and arising from the distribution of the spectra; and a third, the spherical aberration, inversely proportional to the cube of the length.

If we make the length so great that the second and third imperfections are less than the constant one, we have gained all that is possible; thus we see that the exertions of the glass-maker to produce metal of uniform composition enable us, when we are in search of the best possible result, not to shorten but to lengthen the tube.

The investigation into the action of a telescope does not stop at the focus of the principal lens. Properly speaking, no image is formed there; the rays of light go onwards through the eye-piece, and it is only on the retina that there is an image. The action of the eye-lenses is quite as important as that of the object-glass, and it may be possible, by a proper arrangement of the curvatures, to remedy the unavoidable defects already noticed; the eye-piece should be suited specifically to the objective, and the whole instrument has to be treated as an integral system. The second part of the general subject may be treated in a future essay.

Before venturing critically to interpret the appearance of any object as seen through a refractor, the observer would need to know the character of the imperfections of the image. The false disc may be bright in the middle, fading to become imperceptible at the edge, or it may exhibit a coloured fringe; and in all cases the imperfection causes an overlapping of the images of two near objects, giving rise now to the seeming projection of a star upon the moon's disc, then to the phenomenon of Bailey's beads.

The light of an exceedingly faint object diffused over an appreciable disc by one lens may be so faint as to be unperceived, while by another lens, more happily arranged, the brightness may be enough to make the object visible.

An exact knowledge of the indices of the two glasses, and of their curvatures, would enable us to estimate the amount and character of the diffusion; hence the importance of having, along with all lenses of any magnitude, a prism of glass from the same melting, or, better, one cut from a corner which had been left projecting on the rough disc.

6 *Molendo Terrace, Edinburgh,*
1880, *June* 28.

Note on the Nebula near Merope. By Prof. W. Tempel.

(*Extract from a letter to the Foreign Secretary, communicated by Lord Lindsay.*)

The English scientific papers, as also the *Monthly Notices*, have recently directed some attention to the nebula in the neighbourhood of *Merope*, and it is but natural that I should be much interested in anything written or observed in reference to this nebula, which I was the first to discover, on October 19, 1859, when in Venice (*Astr. Nachr.*, No. 1290).

This nebula has already, so to say, a history; for by some its existence has been thoroughly recognised, whilst by others my assertion of it has been contradicted (*Astr. Nachr.*, No. 1393, p. 13).

Amongst the former are to be found various illustrious astronomers, such as Schmidt, Winnecke, Anwers, Schönfeld, &c., who, with instruments of comparatively small size, have attested to the existence of the nebula to the south of *Merope*, and described its form exactly as I saw it from the beginning, and have always continued to see it.

Prof. Schönfeld says in his catalogue, Part II., p. 80: "Anwers 18 = G. Cat. 768. Tempel's nebula near *Merope* very distinct, and immediately conspicuous, even without more accurate indications of its position. Anwers' notices (74) as to its extension and form very correct."

On the other side were ranged astronomers not less distinguished, like D'Arrest, Padre A. Secchi, and the observers at Parsonstown, who failed to see the nebula with their great telescope, and consequently doubted its existence.

But all ambiguity has been since cleared up, for on fitting the large telescopes with eye-pieces of a low magnifying power the nebula becomes distinctly visible, and is shown by them with an image equal in clearness to that given by the smaller instruments.

This fact is no more than is logically to be expected: that what was to be seen with a small telescope should be at least equally visible with a large one. Indeed, any discrepancy ought rather to be the other way, since a small instrument cannot be expected to show all the detail revealed by one of larger size.

It is now ascertained beyond question that the nebula exists. It has been observed with the great Washington Refractor (*Inst. of the Observatory*, p. 45), and again with the Reflector of Lord Rosse (*Observatory*, vol. i., p. 370); and anyone publishing statements as to its non-existence merely uses vain words, and proclaims himself wanting in knowledge of the history of nebulæ and of the management of telescopes.

The description written by Goldschmidt in the year 1864 seems to show that a certain amount of nebulosity surrounds the entire group of the *Pleiades*; but this is an optical illusion, and

in another letter I shall, with your permission, proceed to prove this by the description of a false image in the telescope, a curious phenomenon hitherto unexplained.

My present sketch is, in its main features, the same as that published at Milan (*Pubb. dell' Oss. R. di Brera*, No. 5), only that all the minute stars and the exact form of the nebula have been added here with my *Amici No. I.*, with a magnifying power of 113.

To the right near *Merope* the nebula is so sharply defined that its curvature is clearly traceable, whilst its remaining outline is pale and indistinct. I could only succeed in distinguishing two nuclei, or nodules, and they but little more luminous than the rest.

A glance at the sketch is sufficient to show that in various parts many of the stars are omitted, which, for want of time and owing to pressure of other work, I have as yet been unable to put in. These may amount to some hundreds; indeed, even in the portions which seem thick with stars, many remain to be added, because the scale of the drawing is too small.

Being unable to foresee when it will be in my power to complete a more accurate work, I send this sketch, which is sufficiently so to compare with the drawings of others.

The circle shows the diameter of the field of view of my Steinheil, with an eye-piece of 24.

Comparing my drawing with that of Mr. Maxwell Hall, the two will be found to agree perfectly. That of Mr. Common, on the other hand, has evidently been executed with a telescope of insufficient power to show the *Merope* nebula. This will be obvious to anyone who examines the portions where Mr. Common has drawn three nebulous masses, where minute stars are shown in my sketch; invisible in his telescope as separate points of light, they must have appeared as nebulosity.

My sketch shows such a number of double stars that I have not hitherto been able to find them all in the catalogues, and it is worthy of note that the magnifying power is lower than that ordinarily used in the observation of double stars, showing that *Amici No. I.* gives images so clearly defined as to separate closely-united stars, even when they were not specially looked for, or previously known by me to be thus divisible.

Royal Observatory in Arcetri,
Florence, 1880, May 22.

Observations of Comet I. 1880, *made at the Royal Observatory, Cape of Good Hope.* By David Gill, Her Majesty's Astronomer at the Cape.

The following are the results of observations of Comet I. 1880, made here during the period of the visibility of the Comet. It is a matter of much regret that observations of the nucleus

could not be obtained nearer to perihelion. The view of the Comet was cut off to the south-west by Table Mountain, and the drawings of the tail, made February 2 to February 9, were made at Sea Point, situated to the west of Table Mountain.*

From the amount of haze near the horizon and the small optical power of the portable instrument employed, the nucleus could not be observed at Sea Point, and it was not until February 9, that anything like a nucleus could be made out at the Royal Observatory, and then only by glimpses through cloud.

From February 10 to February 15 (both inclusive) observations were obtained with the $8\frac{1}{2}$ foot Equatoreal (6·9 inches' aperture), but so faint and ill-defined was the object, and so unsatisfactory our instrumental means, that, notwithstanding the greatest care, I do not think the places can be relied upon nearer than within 10″ of arc. On February 10 the parallel-wire micrometer was employed, but afterwards it became necessary to use the ring micrometer.

The observations of the tail of the Comet were made by myself. The observations of the nucleus were made by myself and Mr. Finlay, first assistant—about an equal number of observations being made on each evening by each observer.

The places of the stars of comparison, *a*, *b*, *c*, *d*, *e*, *f*, were determined by early morning meridian observations in the end of June and beginning of July of the present year. It was impossible, from daylight, to obtain meridian observations of these stars sooner. The mean place of the comparison star, $7\frac{1}{2}$ mag., employed by Mr. Ellery on February 14, is also given, derived from meridian observations made in the end of June of the present year.

The corrections of this star to apparent place for February 14 are

$$\text{To R.A.} \quad +0^{s}\cdot52$$

$$\text{N.P.D.} \quad +3''\cdot92$$

Monday, Feb. 2. A small portion only of the tail was seen projecting over south shoulder of Table Mountain. The drawing has no pretension to accuracy.

Tuesday, Feb. 3. This drawing was made from Mr. Henry Solomon's Garden, at Green Point. The night was cloudy, and very hazy, so that the star *β Pis. Aust.* could not be seen, but only guessed at. The only really satisfactory part of the drawing is the position of the tail, as defined by the stars *θ* and *l Gruis*. This is accurately shown for 8.30. At a little past 9 o'clock the star *θ Gruis* was fairly immersed in the tail, and the star *l* was near its southern side.

* See Plate in the April No. facing p. 300.—Ed.

Wednesday, Feb. 4. There is a little doubt about the precise position of the tail relative to β *Pis. Aust.* The star was nearly lost in haze near the horizon, but the position of the tail relative to ϕ *Gruis*, θ *Phœnicis*, τ *Phœn.*, is extremely exact; and relative to η, ξ, and ζ *Phœnicis* is fairly exact—as it there became very faint, but could still be traced beyond α *Hydri* without a definite outline. Epoch $8^h\ 33^m$ Cape M. T.

Thursday, Feb. 5. Owing to haze near the horizon there is a slight uncertainty about the position of the tail relative to γ *Pis. Aust.*, though not much. The position of the tail relative to ε *Phœn.* is very exact. A line joining ε *Phœn.* and μ *Phœn.* is exactly bisected by the northern edge of the tail, whilst ρ *Phœn.* just marked the limit of the southern border of the tail. After this the tail could be traced, as shown, to α *Reticuli*, but the border could not be laid down with great precision. Epoch $8^h\ 40^m$ Cape M. T.

Friday, Feb. 6. The position of the tail very precisely shown relative to δ *Pis. Aust.*, and a small star between δ and Fomalhaut, but owing to haze the position relative to ε *Pis. Aust.* could not be very precisely shown. The position of tail is shown with great accuracy relative to β *Sculp.*, κ and μ *Phœn.* The north border of the tail was well marked by β and δ *Phœn.*, which were only just involved in it. The southern border of the tail seemed to pass half-way between q^1 and q^2, and ϕ *Erid.* and η *Horlog.* were just involved in it. The tail could be traced as shown, without any well-defined boundary passing undoubtedly to N. of α *Retic.* and within α *Doradus* beyond which it was just visible nearly up to *Canopus*. The greater part of the drawing was made at $8^h\ 30^m$, the faint part near *Canopus*, at 9^h.

Saturday, Feb. 7. It is obvious from our present knowledge of the path of the nucleus, that I have been deceived on this date, by the haze and the faintness of the nucleus, into supposing that, as no condensation or nucleus, could be made out, the tail must extend and could be traced as far as ε *Pis. Australis.*

Feb. 8 and 9. The drawings were made by glimpses through cloud, and are both unreliable. That of Feb. 9 is undoubtedly wrong—it would appear that θ has been mistaken for μ Sculptoris.

Observations of Comet I. 1880, *at the Royal Observatory, Cape of Good Hope.* (*Observers, Gill and Finlay.*)

	Cape Mean Time.	Right Ascension.	Parallax Factor.	No. of Obsns.	N.P.D.	Parallax Factor.	No. of Obsns.	Comparison Star.
d	h m s	h m s			° ′ ″			
Feb. 10	8 50 2	0 3 58·59	+0·0665	3				*a*
	8 52 49	,,	...		123 43 15·53	+0·4760	2	*a*
11	8 33 4	0 20 22·16	+0·0661	4	,,	...		*b*
	8 45 42	0 20 31·53	+0·0664	4	123 31 30·78	+0·4434	4	Lacaille 94
12	8 42 18	0 36 28·06	+0·0658	5	123 11 33·93	+0·4145	4	*c*
	8 42 18	0 36 28·29	+0·0658	5	123 11 23·44	+0·4145	5	*d*
13	8 30 57	0 51 39·32	+0·0644	7	122 44 20·93	+0·3888	5	Lacaille 290
14	8 23 5	1 6 7·89	+0·0627	1	122 11 17·23	+0·3435	1	σ Sculptoris
	8 42 51	1 6 19·25	+0·0641	3	122 10 52·86	+0·3806	3	*e*
	8 42 51	1 6 19·79	+0·0641	3	122 10 57·19	+0·3806	3	*f*
15	8 24 31	1 19 54·92	+0·0616	10	121 32 52·17	+0·3340	10	Lacaille 384

The observations are not corrected for Parallax, Mean R.A. and N.P.D. of the stars of comparison.

Star.	R.A. 1880·0	N.P.D. 1880·0	No. of Obsns.	Whence derived.
	h m s	° ′ ″		
a (8 mag.)	0 2 52·80	123 42 17·01	2	Cape Obs. 1880
b (8 mag.)	0 17 34·16	123 31 59·26	3	„ „
Lacaille 94	0 21 58·49	123 40 11·20		Cape 1878, & Melb.
c (8 mag.)	0 34 32·91	123 13 7·58	2	Cape Obs. 1880
d (8 mag.)	0 41 50·65	123 6 48·87	2	„ „
Lacaille 290	0 57 41·58	122 43 25·35		„ 1878
σ Sculptoris	0 56 42·44	122 11 53·71		Cape 1878, & Melb.
e (9 mag.)	1 7 49·07	122 12 37·30	2	Cape Obs. 1880
f (8·7 mag.)	1 11 10·04	122 2 32·98	2	„ „
Lacaille 384	1 17 55·88	121 34 17·09		„ 1878

Mean R.A. and N.P.D. of * observed with Comet at Melbourne on Feb. 14. (7·8 mag.) and $1^h\ 0^m\ 20^s{\cdot}02$. $\Delta = 122°\ 29'\ 46''{\cdot}79$.

On a Method of determining the Pressure on the Solar Surface.

By Prof. E. Wiedemann.

In a paper published in *Wied. Ann.* v., p. 503 (1878), I have shown how experiments on interference bands produced by two rays of light of large difference in phase may give an approximate measure of the mean free path of a molecule in a gas, or rather of the time elapsing between two encounters. I should like to draw the attention of astronomers to the fact that we may in this way obtain information as to the pressure in a luminous gas, whether in our laboratory on the solar surface or in the tail of a comet; for the time elapsing between two encounters of a molecule is almost independent of the temperature, and chiefly depends on the pressure.

In order to make the determination, we must decompose the light sent out by the gaseous body—say a protuberance—by means of the spectroscope, and separate a ray of light, which must be as homogeneous as possible. If we produce Newton's rings between two adjustable pieces of glass and count the number of interference bands which are visible we shall obtain the required information. In the above-mentioned paper I have discussed the number of interference bands seen by J. J. Müller in a sodium flame, and shown that the result agrees well with the

length of the mean free path as calculated according to the kinetic theory of gases. Various methods of producing the interference bands besides the one mentioned may of course be employed.

Note on a Disappearance of Jupiter's Satellites in 1611.
By the Rev. S. J. Johnson.

After calling attention to the fact that on the morning of October 15, 1883, there will be another instance of Jupiter appearing without any satellites exterior to his disc, though for a much shorter period than on the last occasion, August 21, 1867, Mr. Johnson proceeds—'A like instance seems to have occurred to Galileo, unless it arose from want of optical power. On March 15, 1611, after giving the configurations of the satellites at 1h. 30m., he proceeds as follows—"Vix conspici poterant planetæ occidentales. Horâ tandem 3 nullus apparebat, sed omnes, ob maximam vicinitatem cum Jove, latitabant. Postea, ad horam usque 7, multoties Jovem intuitus sum, nullusque planetarum apparuit; ulterius eum jam ad orizontem tendentem non observavi." ' (G. Galilæi, *In Jovis satellites lucubrationes.*)

Abbenhall Rectory,
Mitcheldean, Gloucester, Mar. 3.

Admiral Sir E. Ommanney, writing from Yarmouth, Isle of Wight, August 16, last, records that the luminous bodies called *Perseids* were observed in that locality about the time predicted, August 10, 12, and 13. They were most numerous, August 10, between the hours of 9 P.M. and 11 P.M.; the radiant point was scarcely definable. In general the trajectory of the luminous bodies was short, their visibility only for a brief moment, and the track very attenuated; but a few were large and vivid, one meteor was especially conspicuous which made a traverse from the N.E. to the tail of the Great Bear, expanding into a globular form previous to disappearance. Several meteors were seen at intervals on the night of the 11th, and on the 13th.

Errata.

Page 497, eleventh line from the commencement, *for* 'Double stars and stars with distinct companions' *read* 'Double stars and stars with distant companions."

Page 514, in the measure of Smyth's companion to Sirius, *for* 10″·44 *read* 1044″.

Page 563, tenth line from bottom, last word, *for* 'the' *read* 'ten.'

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